Q: Plot the voltage standing wave patterns for the cases of lossless line terminated with open ($R_L \to \infty$) and lossless line terminated in short ($R_L \to 0$). How are these patterns different from those obtained in class with $R_L > R_0$ & $R_L < R_0$ with $R_L \neq 0$ & $R_L \neq \infty$

Sol: We use the expression for $V(z')$ from notes Eq (1-P52)

(1) $$V(z') = \frac{I_L}{2}(R_L + R_0) e^{j\beta z'} \left\{ 1 + |\Gamma| e^{j(\theta_\Gamma - 2\beta z')} \right\}$$

Under open condition $R_L \to \infty$:

Note that for $R_L \to \infty$ (2) $\Gamma = \dfrac{R_L - R_0}{R_L + R_0} \to$ (3) $1 \angle 0 = 1 e^{j0}$

Also (4) $I_L = 0$ but (5) $V_L \neq 0$ (6) $V_L = I_L R_L$

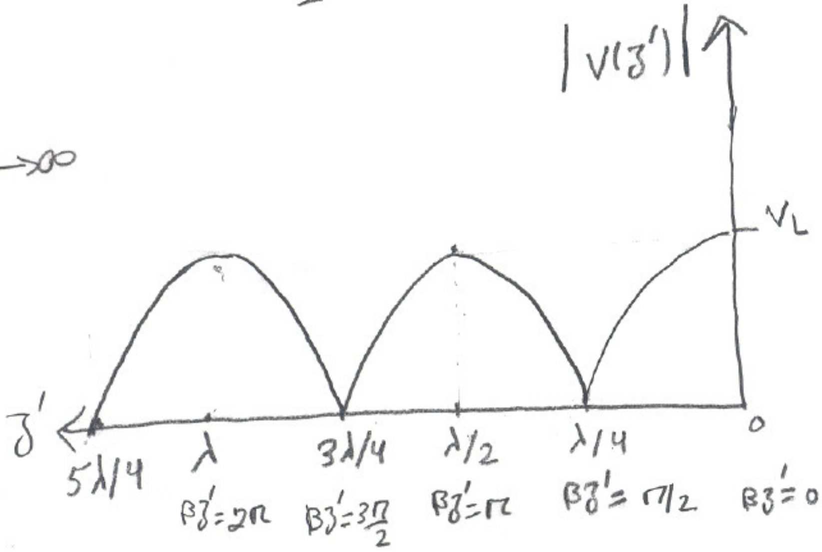
(1) Can be written as

$$V(z') = \frac{V_L + I_L R_0}{2} e^{j\beta z'} \left\{ 1 + e^{-j2\beta z'} \right\} \quad (7)$$

$$= \frac{V_L}{2} \left\{ e^{j\beta z'} + e^{-j\beta z'} \right\} = \boxed{V_L \cos(\beta z')}$$

↑ this is Eq 9-143a of Cheng. You may want to look at the way that Cheng derives this equation.

$R_L \to \infty$



$\beta z' = \frac{2\pi}{\lambda} z'$

Fig 1

* The main difference between this curve & what we found in class for $R_L > R_0$, is that curves in Fig 1 go to zero at $\lambda/4$, $3\lambda/4$, $5\lambda/4$, etc.

Under short condition:

$$R_L \longrightarrow 0 \quad (1) \qquad \Gamma = \frac{R_L - R_0}{R_L + R_0} = -1 = 1e^{-j\pi} \quad (2) \Rightarrow |\Gamma| = 1 \quad (3) \qquad \theta_\Gamma = -\pi \quad (4)$$

For short $V_L = 0$ (5), $I_L \neq 0$ (6) $\quad V_L = I_L R_L$ (7)

Eq (1-P1) $\Rightarrow$

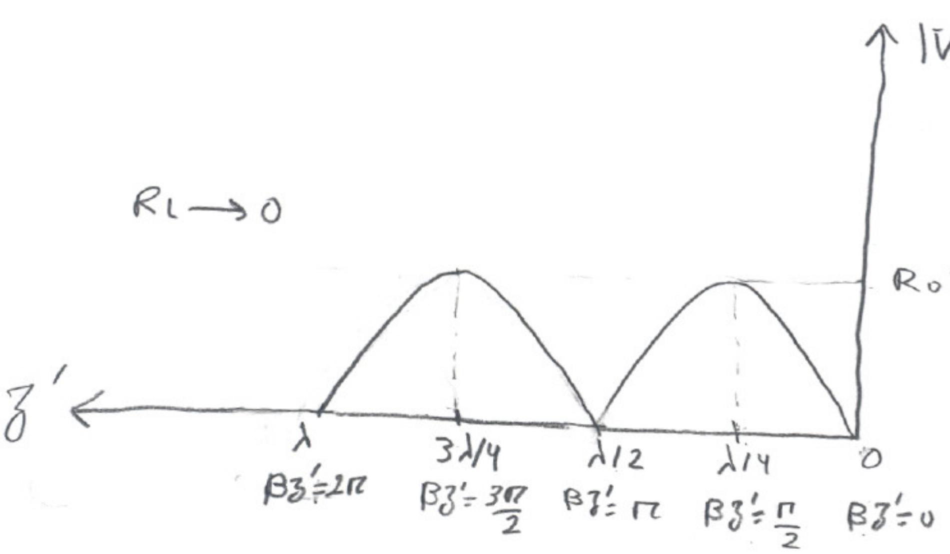
$$(8)\quad V(z') = \frac{\cancel{V_L}^{\,0} + I_L R_0}{2} e^{j\beta z'}\left\{1 + |\Gamma| e^{j(-\pi - 2\beta z')}\right\}$$

$$= \frac{I_L R_0}{2} e^{j\beta z'}\left\{1 - 1\, e^{-j2\beta z'}\right\}$$

$$= \frac{I_L R_0}{2}\left\{e^{j\beta z'} - e^{-j\beta z'}\right\}\frac{j}{j} = \boxed{jI_L R_0 \sin(\beta z')}$$

$$\Rightarrow \boxed{|V(z')| = R_0 \,|I_L \sin(\beta z')|}$$

← this is equation 9-144a of Cheng



The difference between this curve & the one we found in class ($R_L < R_0$) is that here $|V(z')|$ goes to zero at $z' = \lambda/4, 3\lambda/4, \ldots$

Q: The standing wave ratio (S') on a lossless $50\,\Omega$ line ($Z_0 = R_0 = 50$) terminated in an unknown load impedence is found to be 3. The distance between successive voltage minima is 20 (cm), & first minimum is located at 5 (cm) from the load. Determine a) the reflection Coefficient Γ, and b) the load impedence Z_L & c) Find the length of a lossless line and a terminating resistive load that can replace the original load (Z_L) without effecting the waveforms to the left of the Z_L.

Note: this is Example 9-9 of Cheng. repeated here so few important points are brought to you attention.

Sol: We find $|\Gamma|$ from (1) $|\Gamma| = \frac{S'-1}{S+1} = \frac{3-1}{3+1} = \frac{1}{2}$

We also need to find the phase angle for Γ. From you class notes you know that $|V_{min}|$ occurs at (2) $\theta_\Gamma - 2\beta z'_m = -(2n+1)\pi$

$n = 0, 1, 2, \ldots$

where θ_Γ is the Γ Phase angle & z'_m is the location of Voltage minimum measured from the load. since the problem states, the location of first minimum then $n = 0$ in (2) & $z'_m = 5\,\text{cm}$.

But what is β? we are told the distance between successive voltage minima is 20 cm, this means (3) $\frac{\lambda}{2} = 20\,\text{cm} \Rightarrow$

(4) $\lambda = 40\,\text{cm}$ & (5) $\beta = \frac{2\pi}{\lambda} = \frac{2\pi}{0.4} = 5\pi\,(\text{rad})$

then in (2-P3) we have $\theta_r = -\pi + 2\beta z'_m$

$= -\pi + 2 \times 5\pi \times 0.05 = -0.5\pi$ (rad)

① then $\boxed{\Gamma = |\Gamma| e^{j\theta_r} = \frac{1}{2} e^{-j0.5\pi} = 0.5[\cos(\frac{\pi}{2}) - j\sin\frac{\pi}{2}] = -j0.5}$

b) The load impedance can be found from

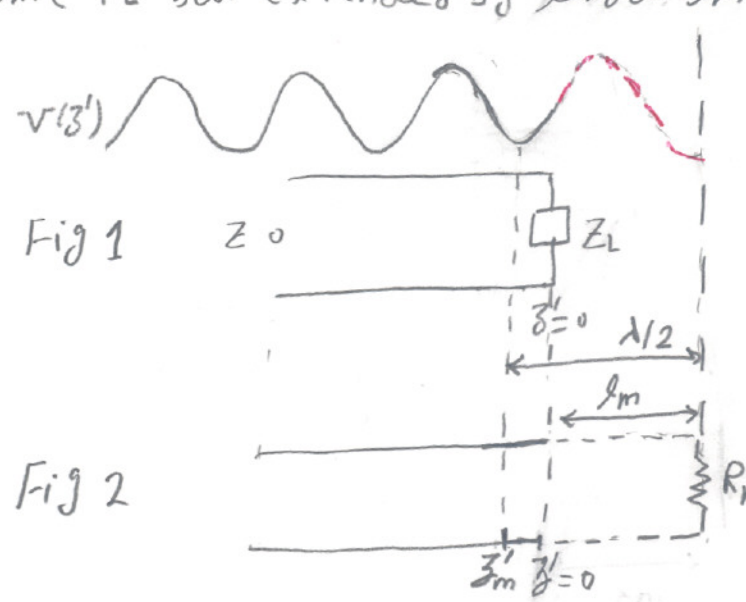
② $\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$ with $Z_0 = 50$ & $\Gamma = -j0.5 \Rightarrow$

$\Gamma(Z_L + Z_0) = Z_L - Z_0 \Rightarrow \Gamma Z_L - Z_L = -\Gamma Z_0 - Z_0 \Rightarrow$

$Z_L(\Gamma - 1) = -Z_0(\Gamma + 1) \Rightarrow \quad Z_L = Z_0 \frac{\Gamma + 1}{1 - \Gamma} \Rightarrow$

$$\boxed{Z_L = 50 \frac{-0.5j + 1}{1 + 0.5j} = 30 - j40}$$

c) To answer the part c it takes a little explanation. Consider Fig. 1 which shows the original TL terminated on Z_L & Fig. 2 which shows the same TL but extended by length l_m & terminated on R_m



* Note the followings about the figures 1 & 2.

1) Neither a voltage maximum nor minimum appears at $z'=0$, since Z_L is arbitrary (here we found out it was $Z_L = 30-j40$) $\underline{\underline{5}}$

2) By extending the TL with length l_m & terminating it on a purely resistive load, we have ensured that either a maximum or minimum will appear at R_m, [see class notes Pages 55-57].
In fact, since $R_L = 30 < R_o = 50$ we will have a minimum at R_m as discussed in lecture notes.

3) The lossless line of length l_m & terminated with R_m must be chosen such that impedance at $z'=0$ & looking to right should be equal to our original Z_L.

4) The impedance at $z'=0$, looking to right can be calculated from

$$Z_i = R_o \frac{R_m + jR_o \tan(\beta l_m)}{R_o + jR_m \tan(\beta l_m)} \quad (1)$$

Eq(1) is the input impedance at $z'=0$ (looking to right) for a lossless line with characteristic impedance of R_o terminated with R_m.

5) The problem [Part (c)] asks us to find l_m & R_m such that the impedance of Eq (1) is the same as Z_L. We found out that $Z_L = 30-j40$, therefor we must solve the following Eq.

$$30-j40 = 50 \frac{R_m + j50\tan(5\pi l_m)}{50 + jR_m \tan(5\pi l_m)} \quad (2)$$

Eq (2-P5) can be sloved for R_m & l_m since there are two equations & two unknowns.

6) There is an easier way to find l_m & R_m. From Figs. 1, 2. Page 4, note that $z'_m + l_m = \lambda/2$. Here we know $z'_m = 0.05$ m & $\frac{\lambda}{2} = 0.2$ meter $\Rightarrow l_m = 0.2 - 0.05 = 0.15$. Also from example 9-8 of Cheng (Page 463), with $R_L < R_o$ we found that

$$R_L = \frac{R_o}{S} \Rightarrow \quad R_m = \frac{R_o}{S} \Rightarrow \quad R_m = \frac{50}{3} = 16.7\ \Omega.$$

7) make sure you study Example 9-8 of Cheng, Page 463.

8) Note that for any load impedance Z_L Eq (1-P5) can be used to find l_m & R_m such that load impedance Z_L can be replaced with a loss less line terminated on R_m, without disturbing the wave forms to the left of $z' = 0$

Q: A lossless TL has capacitance of 200 [pF/m] & inductance of 0.5 [μH/m]. It is excited with sinusoidal source of frequency 1 [KHz]. The magnitude of the voltage measured across a 35 [Ω] load is 100 [V]. Find the following:

a) The line characteristic impedance R_o.
b) The voltage reflection coefficient at load Γ
c) The phase velocity, V_p
d) The wavelength, λ.
e) The forward & backward traveling waves amplitude, V_o^+, V_o^-
c) The line propagation constant β.

sol.:
$$C = 200\times10^{-12}\,[F/m]$$
$$L = 0.5\times10^{-6}\,[H/m]$$
$$f = 1\times10^{3}\,[Hz]$$
$$R_L = 35\,[\Omega]$$
$$|V(z'=0)| = |V(z=l)| = V_L = 100\,[Volt]$$

a) For lossless line $R_o = \sqrt{\frac{L}{C}} = \sqrt{\frac{0.5\times10^{-6}}{200\times10^{-12}}} = \boxed{50\,[\Omega]}$

b) $\Gamma = \frac{R_L - R_o}{R_o + R_L} = \frac{35-50}{35+50} = \boxed{-0.1765}$

c) $V_p = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{LC}} = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.5\times10^{-6}\times200\times10^{-12}}} = \boxed{1\times10^{8}} = 0.33\,c\ [m/s]$

speed of light in vacuum

d) $\lambda = \frac{V_p}{f} = \frac{1\times10^8}{1\times10^3} = \boxed{10^5\ [m]}$ (1) Note this is 1/3 of the operating wavelength

e) Recall (2) $V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}$

$$= V_0^+ e^{-j\beta z}\left[1 + \frac{V_0^- e^{j\beta z}}{V_0^+ e^{-j\beta z}}\right]$$

(3) at load $z = l$ we have

(4) $V(z=l) = V_L = V_0^+ e^{-j\beta l}\left[1 + \frac{V_0^- e^{j\beta l}}{V_0^+ e^{-j\beta l}}\right] = V_0^+ e^{-j\beta l}[1+\Gamma]$

where $\Gamma = \Gamma_L = \frac{V_0^- e^{j\beta l}}{V_0^+ e^{-j\beta l}}$ (5) $\quad$ (4) $\Rightarrow$ (6) $|V_L| = |V_0^+|\,|1+\Gamma| \Rightarrow$

(7) $|V_0^+| = \frac{|V_L|}{|1+\Gamma|} = \frac{100}{|1-0.1765|} \Rightarrow \boxed{|V_0^+| = 121.4286 \text{ Volt}}$

similarly from (2) we have $V_L = V_0^- e^{j\beta l}\left[1 + \frac{V_0^+ e^{-j\beta l}}{V_0^- e^{j\beta l}}\right]$

$$= V_0^- e^{j\beta l}\left[1 + \frac{1}{\Gamma}\right]$$

$$= V_0^- e^{j\beta l}\left[\frac{\Gamma+1}{\Gamma}\right] \Rightarrow$$

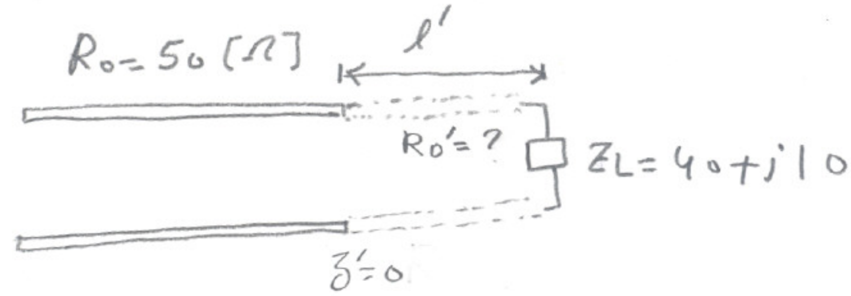
$|V_L| = |V_0^-|\,\left|\frac{\Gamma+1}{\Gamma}\right| \Rightarrow |V_0^-| = \left|\frac{\Gamma}{\Gamma+1}\right||V_L| = \left|\frac{-0.1765}{1-0.1765}\right| \times 100 \Rightarrow$

$\boxed{|V_0^-| = 21.4286\ [\text{Volt}]}$

c) $\beta = \frac{2\pi}{\lambda} = \frac{2\pi}{10^5} = 6.2832\times10^{-5}$ [rad/m]

Q: A Transmission line of characteristic impedance $R_0 = 50\ [\Omega]$ is to be matched to a load impedance $Z_L = 40 + j10\ [\Omega]$ through a length l' of another transmission line of characteristic impedance R_0'. what are the required l' & R_0' for matching.

Sol:



We calculate the input impedance seen at $z' = 0$ & looking to the right (toward the load Z_L). To have match condition this input impedance must be made equal to the original line impedance, i.e. $R_0 = 50\ \Omega$.

① $$Z_i = R_0' \frac{Z_L + jR_0' \tan(\beta l')}{R_0' + jZ_L \tan(\beta l')}$$ let $\tan \beta l' = T$

input at $z' = 0$ & to the right

② $$Z_i = R_0' \frac{Z_L + jR_0' T}{R_0' + jZ_L T}$$

Now as stated earlier to have matched condition we must equate $Z_i = 50$, which is the original line impedance - substituting values for Z_i, & Z_L we have

$$50 = R_o' \frac{(40+10j) + jR_o' T}{R_o' + j(40+10j)T} \Rightarrow$$

$$50\{R_o' + (40j-10)T\} = 40R_o' + j(10R_o' + R_o'^2 T)$$

$$50R_o' - 500T + j2000T = 40R_o' + jR_o'(10 + R_o'T) \Rightarrow$$

$$50R_o' - 500T = 40R_o' \Rightarrow \boxed{50R_o' = 40R_o' + 500T} \quad ①$$

&

$$2000T = 10R_o' + (R_o')^2 T \Rightarrow \boxed{-(R_o')^2 T = 10R_o' - 2000T} \quad ②$$

solve (1) & (2) for R_o' & T & we have

③ $R_o' = 38.7$ & ④ $T = \tan(\beta l') = 0.775 \Rightarrow$

$$\tan\left(\frac{2\pi}{\lambda} l'\right) = 0.775 \Rightarrow \boxed{l' = 0.105\lambda}$$

Q: A generator with an open ckt voltage $V_g = 10\cos 8000\pi t$ [V] & internal impedance of $Z_g = 40 + j30$ is connected to a 50[Ω] distortionless line. The line has a resistance of 0.5 [Ω/m] & its lossy dielectric medium has a loss tangent of 0.18%. The line is 50(m) long & is terminated in a matched load. Find (a) the instantaneous expressions for the voltage & current at an arbitrary location on the line (b) the instantaneous expressions for the voltage & current at the load (c) the average power transmitted to the load

Sol: Let us write down the information given

$V_g = 10\angle 0$, $\omega = 8000\pi$, $Z_g = 40 + j30$, $R = 0.5$ [Ω/m]

$Z_0 = R_0 = 50$, $Z_L = R_0 = 50$ (matched condition)

① $\tan\delta_c = \dfrac{\varepsilon''}{\varepsilon'} = 0.0018$, ② since this is a distortionless line $\dfrac{R}{L} = \dfrac{G}{C}$.

$l = 50$ [m], &

For line that is matched ③ $V(z) = V_i e^{-\gamma z}$ &

④ $I(z) = I_i e^{-\gamma z}$

We need to calculate ⑤ $\gamma = \alpha + j\beta$. For distortionless line

⑥ $\alpha = R\sqrt{\dfrac{C}{L}}$ ⑦ $\beta = \omega\sqrt{LC}$. To find α & β

we need to know C & L

From Eq. for loss tan ① $\tan\delta_c = \frac{\varepsilon''}{\varepsilon'} \simeq \frac{\sigma}{\omega\varepsilon}$ (See Cheng Eq 7-115 page 342)

Also recall that in previous $\underbrace{\text{HW}}_{}$ we showed (Set #3 Prob 5)

that ② $\frac{\sigma}{\varepsilon} = \frac{G}{C}$ & since for distortionless line ③ $\frac{G}{C} = \frac{R}{L}$ we can

write ④ $\tan\delta_c = 0.0018 = \frac{\varepsilon''}{\varepsilon'} \simeq \frac{\sigma}{\omega\varepsilon} = \frac{G}{\omega C} = \frac{1}{\omega}\frac{R}{L} \Rightarrow$

$$L = \frac{R}{\omega \tan\delta_c} = \frac{0.5}{8000\pi \times 0.0018} = \boxed{0.0111\ [H/m]}$$

* for distortionless line $R_0 = \sqrt{\frac{L}{C}} \Rightarrow C = \frac{L}{R_0^2} = \frac{0.0111}{(50)^2} \Rightarrow$

$$C = 4.4210 \times 10^{-6} = \boxed{4.4210\ [\mu F/m]}$$

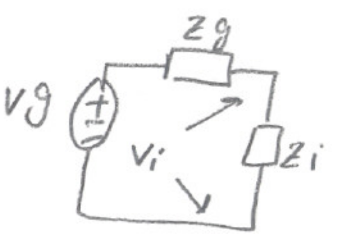
then from (6-P11) $\alpha = R\sqrt{\frac{C}{L}} = 0.5\sqrt{\frac{4.421\times10^{-6}}{0.0111}} \Rightarrow$

$$\boxed{\alpha = 0.01\ [NP/m]}$$

From (7-P11)

$$\boxed{\beta = \omega\sqrt{LC} = 8000\pi\sqrt{0.0111 \times 4.421\times10^{-6}} = 5.5556\ [rad/m]}$$

* To use Eqs (3-P11) $V(z) = V_i e^{-\alpha z}$ we also need to know V_i. Recall that V_i can be calculated from voltage divider



$$V_i = V_g \frac{Z_i}{Z_i + Z_g}$$

& for matched line $Z_i = Z_0$ & here $Z_0 = R_0 \Rightarrow$ $Z_i = Z_0 = R_0 = 50$ & $Z_g = 40 + j30$

$$V_i = \frac{10\angle 0 \times 50}{50 + 40 + j30} \Rightarrow$$

$$V_i = \frac{500\angle 0}{90+j30} = \frac{50\angle 0}{9+j3} = \frac{50\angle 0}{9.4868\angle 0.3218} \Rightarrow$$

① $V_i = 5.2705\angle -0.3218$ then from (3-P11) we have

$$V(z) = V_i\, e^{-\gamma z} = V_i\, e^{-\alpha z}\, e^{-j\beta z}$$

$$V(z) = 5.2705\angle -0.3218\; e^{-0.01z}\, e^{-j5.5556z}$$

$$= 5.2705\, e^{-0.01z}\, e^{-j(5.5556z+0.3218)} \Rightarrow$$

$$v(z,t) = \mathrm{Re}[V(z)e^{j\omega t}] = \mathrm{Re}\left[5.2705\, e^{-0.01z}\, e^{j[8000\pi t - 5.5556z - 0.3218]}\right]$$

①

$$\boxed{v(z,t) = 5.2705\, e^{-0.01z}\cos(8000\pi t - 5.5556z - 0.3218)}$$

For $I(z) = I_i e^{-\gamma z}$ we note that $I_i = \frac{V_g}{Z_g+Z_i} = \frac{Z_i}{Z_i+Z_g} V_g \frac{1}{Z_i}$

$$= \frac{V_i}{Z_i} = \frac{V_i}{Z_0}$$ since for matched line $Z_i = Z_0$

then

$$I(z) = \frac{V_i\, e^{-\gamma z}}{Z_0} \Rightarrow$$

$$\boxed{I(z) = 0.1054\, e^{-0.01z}\cos(8000\pi t - 5.5556z - 0.3218)}$$

b) at load $z = 50 \Rightarrow$

$$v_L = v(z=50,t) = 5.2705\, e^{-0.01\times 50}\cos(8000\pi t - 5.5556\times 50 - 0.3218)$$

$$= 3.2\cos(8000\pi t - 278.102)$$

$$i_L = i(z=50,t) = 0.064\cos(8000\pi t - 278.102)$$

c) $P_{ave} = \frac{1}{2} Re[V I^*]$

$$= \frac{1}{2} Re\left[5.2705\, e^{-0.01\times 50}\, e^{-j(5.5556\times 50+0.3218)} \times 0.1054 \times e^{-0.01\times 50}\, e^{+j(5.5556\times 50+0.3218)}\right]$$

$$= \frac{1}{2}\, 5.2705 \times 0.1054 \times e^{-2\times 0.01\times 50}$$

$$= 0.102 \text{ Watt}$$

Q: A 2 [m] lossless air-spaced transmission line having a characteristic impedence 50 [Ω] is terminated with an impedence 40 + j 30 (Ω). At an operating frequency of 200 [MHZ]. the phase velocity in the line is the same as speed of light in Vacuum (C). What is the line input impedance

Sol: $l = 2[m]$, $Z_0 = R_0 = 50\,[\Omega]$ lossless, $Z_L = 40 + j30$

$f = 200\times 10^6$ [HZ], $V_p = C = 3\times 10^8$

$$Z_i = R_i + j X_i = R_0 \frac{Z_L + j R_0 \tan(\beta l)}{R_0 + j Z_L \tan(\beta l)}$$

$$V_p = \frac{\omega}{\beta} \Rightarrow \beta = \frac{\omega}{V_p} = \frac{\omega}{c} = \frac{2\pi \times 200\times 10^6}{3\times 10^8} = 4.189 \text{ [rad/m]}$$

$$\beta l = 4.189 \times 2 = 8.378 = 480^\circ$$

$$Z_i = 50 \frac{(40+j30) + j\,50\tan(8.378)}{50 + j(40+j30)\tan(8.378)} \Rightarrow$$ 15

$$Z_i = 26.32 - j\,9.87$$

Q: A sinusoidal voltage generator with $V_g = 0.1\angle 0$ [V] & internal impedance $Z_g = R_0$ is connected to a lossless transmission line having a characteristic impedance $R_0 = 50$ [Ω]. The line is l meter long & is terminated in a load resistence $R_L = 25$ [Ω].

Find a) V_i, I_i, V_L & I_L in terms of the line length (l).

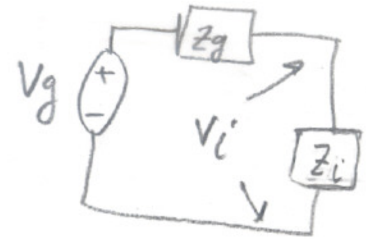
b) The standing-wave ratio on the line & c) the average power delivered to the load.

Sol: $V_g = 0.1\angle 0$; $Z_g = R_g = R_0 = 50$, line is lossless & is matched to the generator. ($Z_L = R_L = 25$ (the line is not matched to the load))

a) For a lossless line $Z_i = R_0 \dfrac{R_L + jR_0\tan(\beta l)}{R_0 + jR_L\tan(\beta l)}$

with $R_0 = 50$, $R_L = 25$ $\Rightarrow$

$$\boxed{Z_i = 50 \frac{1 + 2j\tan(\beta l)}{2 + j\tan(\beta l)}}$$



① $$V_i = V_g \frac{Z_i}{Z_i + Z_g}$$

② $$I_i = \frac{V_g}{Z_i + Z_g} = \frac{V_i}{Z_i}$$

(1) ⇒ $$V_i = 0.1 \frac{50\,\frac{1+2j\tan(\beta l)}{2+j\tan(\beta l)}}{50\,\frac{1+2j\tan(\beta l)}{2+j\tan(\beta l)} + 50} = \boxed{\frac{1}{30}\,\frac{1+j2\tan(\beta l)}{1+j\tan(\beta l)}}$$

$$I_i = \frac{V_i}{Z_i} = \frac{\frac{1}{30}\,\frac{1+j2\tan(\beta l)}{1+j\tan(\beta l)}}{50\,\frac{1+2j\tan(\beta l)}{2+j\tan(\beta l)}} = \boxed{\frac{2+j\tan(\beta l)}{1500[1+j\tan(\beta l)]}\ [A]}$$

or we can write I_i in [mA] by

$$I_i = 1000 \times \frac{2+j\tan(\beta l)}{1500[1+j\tan(\beta l)]} = \boxed{\frac{2}{3}\,\frac{2+j\tan(\beta l)}{1+j\tan(\beta l)}\ [mA]}$$

* $$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{R_L - R_0}{R_L + R_0} = \frac{25-50}{25+50} = -\frac{1}{3} = \frac{1}{3} e^{j\pi} = \frac{1}{3}\angle \pi$$

* To Find V_L & I_L we use Eqs (9-135a) & (9-135b) of Cheng.

$$V(z') = \frac{Z_0 V_g}{Z_0 + Z_g} e^{-\gamma z} \left(\frac{1 + \Gamma_L e^{-2\gamma z'}}{1 - \Gamma_g \Gamma_L e^{-2\gamma l}} \right) \quad ①$$

where $\Gamma_L = -\frac{1}{3}$ & ② $\Gamma_g = \frac{Z_g - Z_0}{Z_g + Z_0} = \frac{R_g - R_0}{R_g + R_0} = \frac{50-50}{50+50} = 0$

For lossless line & when $\Gamma_g = 0$, (1) simplifies to

$$V(z') = \frac{R_0 V_g}{R_0 + R_g} e^{-j\beta z} \left(1 + \Gamma_L e^{-2j\beta z'} \right) \quad ②$$

* Furthermore at load $z' = 0 \doteq z = \ell$, then (2-P16) $\Rightarrow$

$$V_L = V(z'=0)\big|_{z=\ell} = \frac{R_0 V_g}{R_0 + R_g} e^{-j\beta\ell}(1+\Gamma_L) \Rightarrow$$

$$V_L = \frac{50 \times 0.1}{50+50} e^{-j\beta\ell}\left(1-\frac{1}{3}\right) = \boxed{\frac{1}{30} e^{-j\beta\ell}}$$

$$I_L = \frac{V_g}{R_0+R_0} e^{-j\beta\ell}(1-\Gamma') = \frac{0.1}{100} e^{-j\beta\ell}\left(1+\frac{1}{3}\right) = \frac{4}{3000} e^{-j\beta\ell} \ [A]$$

or

$$I_L = 1000 \, \frac{4}{3000} e^{-j\beta\ell} = \boxed{\frac{4}{3} e^{-j\beta\ell} \ [mA]}$$

b) $$S' = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+1/3}{1-1/3} = 2$$

c) $$(P_{load})_{\substack{time \\ averaged}} = \frac{1}{2} Re\left[V_L I_L^*\right] = \frac{1}{2} Re\left[\frac{1}{30} \times \frac{4}{3000}\right]$$

$$\boxed{= \frac{2}{90000} = 2.22 \times 10^{-5} \ [Watt]}$$