HW 4

Q: let $Z_L = 60 + j\,43\,[\Omega]$ & $Z_0 = 50\,[\Omega]$. Find the input impedance with $l = 0.32\,[m]$ & line wavelength of $\lambda = 0.854\,[m]$.

Sol: we first calculate the normalized load impedance

$$z_l = \frac{Z_L}{Z_0} = \frac{60 + j\,43}{50} = 1.2 + j\,0.86 = r + jx$$

locate $z_l = 1.2 + j\,0.86$ on the chart, this is point 1 on the figure.

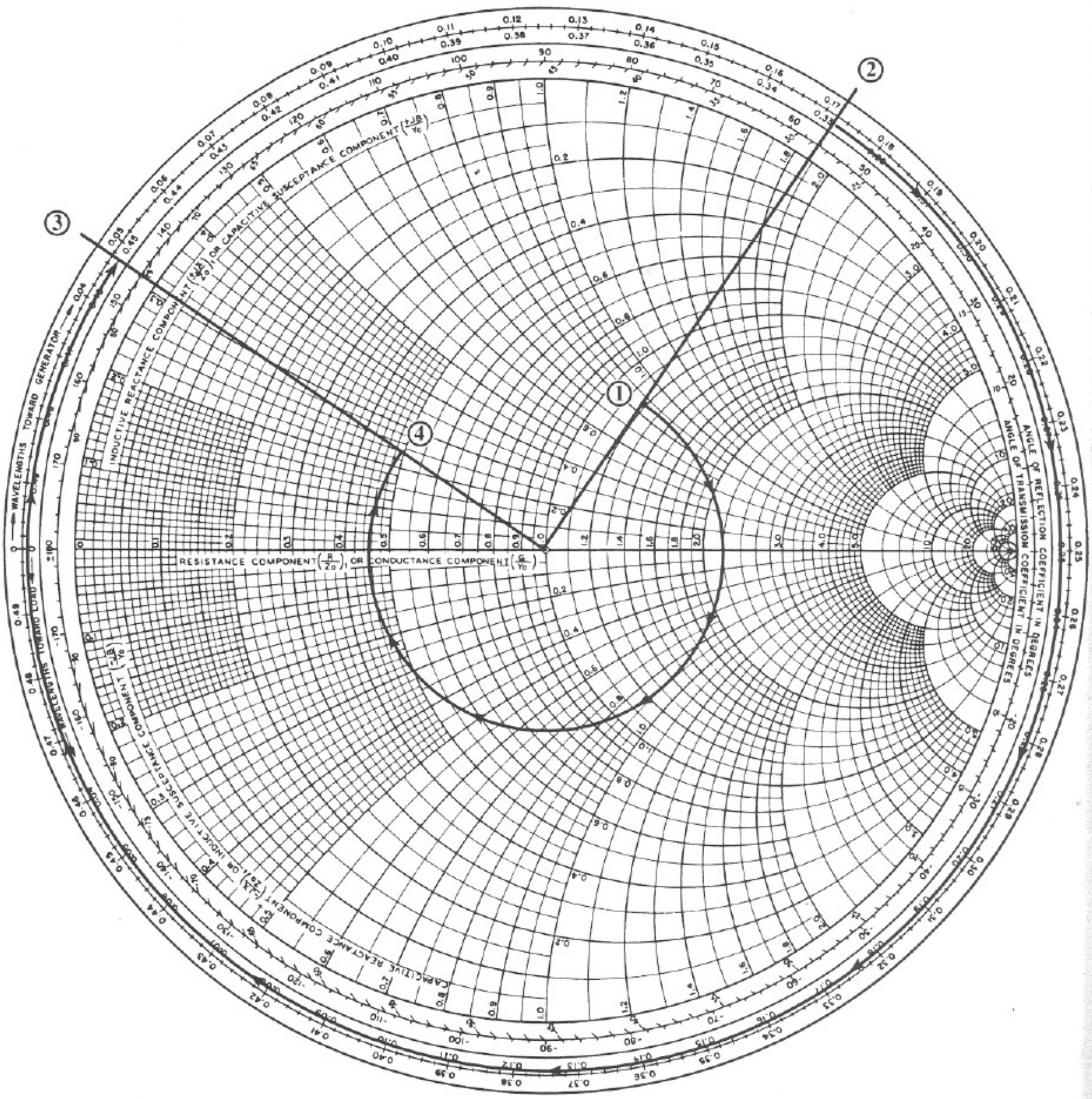
* To find the input impedance a distance $l = 0.32$, we first evaluate

$$\frac{\Delta z'}{\lambda} = \frac{l}{\lambda} = \frac{0.32}{0.854} = 0.375$$

* Join 1 to perimeter via line 1 - 2. Read the value for $\frac{l}{\lambda}$ at 2 which is ≈ 0.172. Now move from 2 toward the generator a distance equal to 0.375. In other words you need to locate a point for which we have $0.172 + 0.375 = 0.547$. This means we go around the perimeter, come to $\frac{z'}{\lambda} = 0.5$ (this is P_{sc}) & go further by an amount $= 0.547 - 0.5 = 0.047$. This point is marked 3.

* Draw a line from 3 to center. This line intersects the original circle at 4. The r & x for point 4 are given by

$$z_i = 0.5 + j\,0.25 \Rightarrow Z_i = 50\,z_i = 25 + j\,12.5\,[\Omega]$$


1
2
3
4
WAVELENGTHS TOWARD GENERATOR
WAVELENGTHS TOWARD LOAD
ANGLE OF REFLECTION COEFFICIENT IN DEGREES
ANGLE OF TRANSMISSION COEFFICIENT IN DEGREES
RESISTANCE COMPONENT (R/Zo), OR CONDUCTANCE COMPONENT (G/Yo)
INDUCTIVE REACTANCE COMPONENT (+jX/Zo), OR CAPACITIVE SUSCEPTANCE COMPONENT (+jB/Yo)
CAPACITIVE REACTANCE COMPONENT (-jX/Zo), OR INDUCTIVE SUSCEPTANCE COMPONENT (-jB/Yo)

Q: The inductance & capacitance of a lossless 50[Ω] line are 0.251 [μH/m] & 99.5 [pF/m]. The line is attached to a source of $10\cos(2\pi 10^6 t)$ with internal impedance of 1 [Ω]. The length of the line is 5 meter & the is terminated on a load resistance of 50[Ω]. a) What are the instantaneous voltage & current at any point b) what is the power delivered to the load

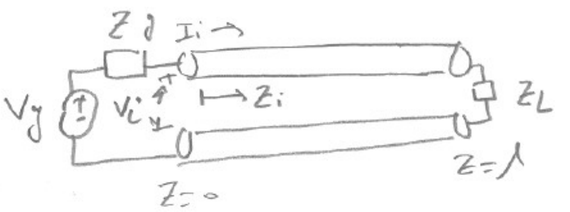
Sol: $Z_0 = R_0 = 50$, $L = 0.251\times10^{-6}$ [H/m], $C = 99.5\times10^{-12}$ [F/m],

$\omega = 2\pi\times10^6$, $Z_g = R_g = 1$, $l = 5$, $Z_L = R_L = 50$, $V_g = 10\angle 0$

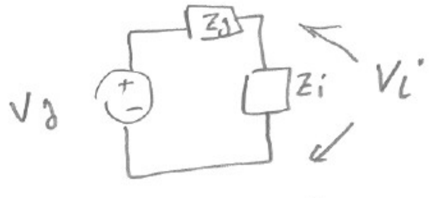
* since the line is match there is no reflected wave & the voltage & current (in phasor form) are given by

① $V(z) = V_i\, e^{-j\beta z}$

② $I(z) = I_i\, e^{-j\beta z}$



* To find $V(z)$ & $I(z)$ we need to find I_i & V_i. These are given by Eq(3,4),



but what is Z_i. For matched line $Z_i = Z_0$ since

③ $V_i = V_g \dfrac{Z_i}{Z_i + Z_g} =$

④ $I_i = \dfrac{V_g}{Z_g + Z_i}$

$$Z_i = R_0 \frac{Z_L + jR_0\tan(\beta l)}{R_0 + jZ_L\tan(\beta l)} =$$

⑤ $$R_0 \frac{R_0 + jR_0\tan(\beta l)}{R_0 + jR_0\tan(\beta l)} = R_0 = Z_0$$

* Then ① $V_i = V_g \dfrac{Z_o}{Z_o + Z_g} = 10 \dfrac{50}{50+1} = 9.8039 \angle 0$ 4

② $I_i = \dfrac{V_g}{Z_g + Z_o} = \dfrac{10}{1+50} = 0.1961 \angle 0$

* β cense found from ③ $\beta = \dfrac{\omega}{V_p}$ & ④ $V_p = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{0.251\times10^{-6} \times 99.5\times10^{-12}}}$ ⇒

⑤ $V_p = 2\times10^8$ [m/s] & $\beta = \dfrac{2\pi\times10^6}{2\times10^8} = \boxed{0.01\pi}$

* We now have enough information to write $V(z)$ & $I(z)$

⑥ $V(z) = 9.8039\, e^{-j0.01\pi z}$

⑦ $I(z) = 0.1961\, e^{-j0.01\pi z}$ ⇒

$v(z,t) = Re[V(z) e^{j\omega t}] = 9.8039 \cos(\omega t - 0.01\pi z)$

$i(z,t) = Re[I(z) e^{j\omega t}] = 0.1961 \cos(\omega t - 0.01\pi z)$

b) For a lossless line terminated on a match Power delivered to the load is the same as power input to the line

$(P_{ave})_L = (P_{ave})_i = \frac{1}{2} Re[V_i \cdot I_i^*] = \frac{1}{2}\, 9.8039 \times 0.1961$

$= 0.9613$ [watt]

Q: Let P_i be the incident time averaged power approaching a point on a lossless line, P_r the time averaged reflected power on the line, and P_t the time averaged transmitted power available to do work on the load. You can think of the P_t as a useful power since, for example it can be radiated by an antenna connected to the line. Show the following is true $P_t = P_i - P_r = \frac{|V_0^+|^2}{2Z_0}(1-|\Gamma_L|^2) = P_i(1-|\Gamma_L|^2)$

Sol: The voltage & current on line are given by (lossless case)

① $V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}$

② $I(z) = I_0^+ e^{-j\beta z} + I_0^- e^{j\beta z} = \frac{V_0^+}{Z_0} e^{-j\beta z} - \frac{V_0^-}{Z_0} e^{j\beta z}$

or equally

③ $V(z) = V_0^+ e^{-j\beta z}\left[1 + \frac{V_0^- e^{j\beta z}}{V_0^+ e^{-j\beta z}}\right]$

④ $I(z) = \frac{V_0^+}{Z_0} e^{-j\beta z}\left[1 - \frac{V_0^- e^{j\beta z}}{V_0^+ e^{-j\beta z}}\right]$

since $z + z' = l \Rightarrow z = l - z'$ we can write (3) & (4) as

⑤ $V(z) = V_0^+ e^{-j\beta z}\left[1 + \frac{V_0^- e^{j\beta l} e^{-j\beta z'}}{V_0^+ e^{-j\beta l} e^{+j\beta z'}}\right]$

⑥ Recall $\Gamma(z) = \frac{V_0^- e^{j\beta z}}{V_0^+ e^{-j\beta z}}$ & ⑦ $\Gamma(z=l) = \Gamma_L = \frac{V_0^- e^{j\beta l}}{V_0^+ e^{-j\beta l}}$

then (5) $\Rightarrow$

⑦ $\boxed{V(z) = V_0^+ e^{-j\beta z}\left[1 + \Gamma_L e^{-2j\beta z'}\right]}$ &

similarly

⑧ $\boxed{I(z) = \frac{V_0^+}{Z_0} e^{-j\beta z}\left[1 - \Gamma_L e^{-2j\beta z'}\right]}$

Now the time averaged Power transmitted at any point is

① $$P_t = \frac{1}{2} Re[V(z) I^*(z)]$$

$$= \frac{1}{2} Re\left[V_0^+ e^{-j\beta z}(1+\Gamma_L e^{-2j\beta z'}) \cdot \frac{(V_0^+)^*}{Z_0^*} e^{j\beta z}[1-\Gamma_L^* e^{2j\beta z'}]\right]$$

$$= \frac{1}{2} Re\left[\frac{|V_0^+|^2}{Z_0^*}(1-\Gamma_L^* e^{2j\beta z'} + \Gamma_L e^{-2j\beta z'} - |\Gamma_L|^2)\right]$$

* Recall ② $V_0^+ \cdot (V_0^+)^* = |V_0^+|^2$ & ③ $\Gamma_L \Gamma_L^* = |\Gamma_L|^2$

(1) ⇒ ④ $$P_t = \frac{1}{2} Re\left[\frac{|V_0^+|^2}{Z_0}(1+\Gamma_L e^{-2j\beta z'} - (\Gamma_L e^{-2j\beta z'})^* - |\Gamma_L|^2)\right]$$

since ⑤ $Z_0 = Z_0^*$ because the line is lossless.

but ⑥ $\Gamma_L e^{-2j\beta z'} - (\Gamma_L e^{-2j\beta z'})^* = j2\, Im[\Gamma_L e^{-2j\beta z'}]$

then

⑦ $$P_t = \frac{1}{2} Re\left[\frac{|V_0^+|^2}{Z_0}(1+2j\, Im[\Gamma_L e^{-2j\beta z'}] - |\Gamma_L|^2)\right]$$

⑧ $$\boxed{P_t = \frac{1}{2}\left[\frac{|V_0^+|^2}{Z_0}(1-|\Gamma_L|^2)\right] = \frac{1}{2}\frac{|V_0^+|^2}{Z_0}(1-|\Gamma_L|^2)}$$

* We can think of $\frac{|V_0^+|^2}{2Z_0}$ as the power incident & $\frac{|V_0^+|^2}{2Z_0}|\Gamma_L|^2$ as the power reflected then

$$P_t = \frac{1}{2}\frac{|V_0^+|^2}{Z_0}(1-|\Gamma_L|^2) = P_i - P_r = P_i(1-|\Gamma_L|^2)$$

So to have maximum power available for work we should minimize $|\Gamma_L|^2 \Rightarrow \Gamma_L = 0 \Rightarrow S = 1$ (matched condition)

Q: The capacitance of a 0.6 [m] long lossless line measured at 100 [KHz] was 54 [PF] & its inductance was equal to 0.3 [MH].

a) Determine Z_o

b) Calculate X_{io} & X_{is} (open & short ckt impedence) at 10MHz

c) Can you find out the value of the dielectric constant of the cable insulating medium

sol: a) $C' = \dfrac{54\times10^{-12}}{0.6} = 9\times10^{-11}$ [F/m]

$L = \dfrac{0.3\times10^{-6}}{0.6} = 5\times10^{-7}$ [H/m]

$Z_o = R_o = \sqrt{\dfrac{L}{C}} = \sqrt{\dfrac{5\times10^{-7}}{9\times10^{-11}}} = 74.54\ [\Omega]$

b) $Z_{io} = -jX_{io} = -j R_o \cot g\, \beta l$; $\beta = \omega\sqrt{LC} = 2\pi\times10^{7}\sqrt{\dfrac{0.3\times10^{-6}}{0.6}\times\dfrac{54\times10^{-12}}{0.6}}$

$Z_{io} = jX_{io} = -j\, 74.54 \cot g(0.4215\times0.6)$ $\beta = 0.4215$

$= -j\, 288.42$

$Z_{is} = j R_o \tan(\beta l) = j\, 74.54 \tan(0.4215\times0.6) = j\, 19.26$

c) To do this part we need a piece of information that although we did not cover in class, yet it is very useful.

$\mu\varepsilon = LC' \Rightarrow \mu_r\mu_o\varepsilon_r\varepsilon_o = LC \rightarrow$ since $\mu_r = 1$ &

$\varepsilon_r = LC'c^2 = 9\times10^{-11}\times5\times10^{-7}\times(3\times10^{8})^2 = 4.05$ $\quad c = \dfrac{1}{\sqrt{\mu_o\varepsilon_o}}$

speed of Light in vacuum

speed of light in vacuum

Q: The characteristic impedance of a given losslen transmission line is 75 [Ω]. Use a smith chart to find the impedance & admittance at 200 (MHz) of such line that is (a) 1 [m] long & open circuited, and (b) 0.8 [m] long & short circuited. 8

Sol: $R_0 = 75$

a) $l = 1$ $\lambda = \frac{c}{v} = \frac{3\times10^8}{200\times10^6} = 1.5$ [m]

$\frac{z'}{\lambda} = \frac{1}{1.5} = 0.667.$

Since open ckt, enter the chart at P_{oc} (the point on the extreme right). The value of $\frac{z'}{\lambda}$ here is 0.25. To find input impedance move toward the generator an amount equal to 0.667 i.e. the total distance of 0.25 + 0.667 = 0.917. To find the 0.917 on the smith chart perimeter, marked as wavelength toward generator, note that you must start from $\frac{z'}{\lambda} = 0$ which is on the extrem left.

Going around one time is equal to 0.5, then 0.917 − 0.5 = 0.417.

locate 0.417 on the wavelength toward generator. This is marked as P_1. Draw a line from P_1 to 0 & continue to. OP_1 intersects the x-circles at $x \simeq 0.57$ then $\boxed{Z_i = -j\,0.57\times75 = -j\,42.75}$

The line OP_2 intersects the x-circles at $x \simeq 1.75$ then

$$\boxed{Y_i = j\,\frac{1.75}{75} \simeq j\,0.023}$$

b) $l = 0.8 \Rightarrow \frac{\Delta z'}{\lambda} = \frac{0.8}{1.5} = 0.533$

Enter the chart at P_{sc} (extreme let). Move toward generator a distance 0.533. This means going around one time & then another 0.033. ($0.533 = 0.5 + 0.033$). This point is marked P_3.

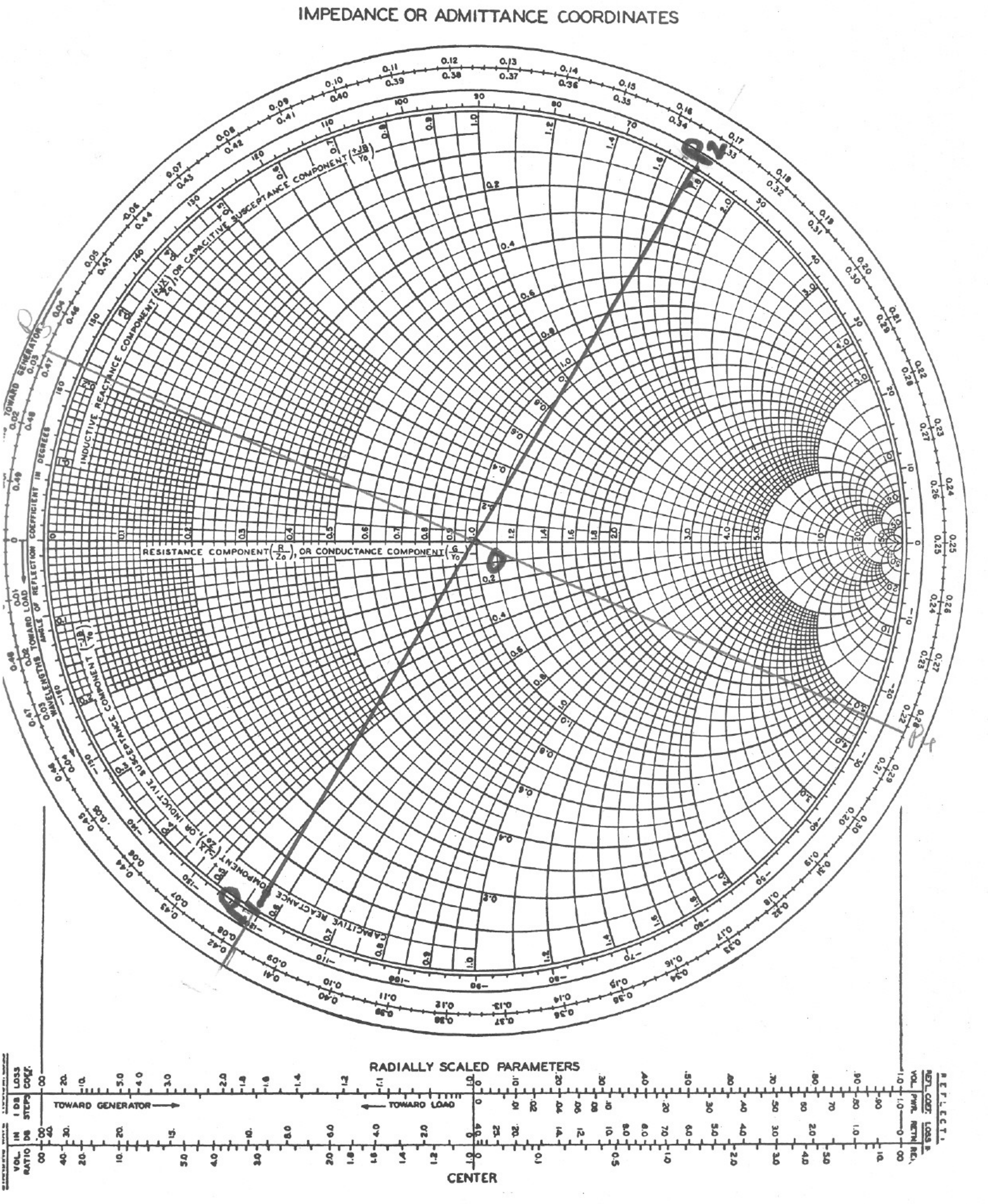
Draw a line Joining P_3 to O & extend it to the other side (to P_4).

Read the value of x-circle intersecting OP_3 & $r=0$ circle.

This the input impedance of the shorted line.

$x = 0.21 \Rightarrow Z_i = jx R_o = j\,0.21 \times 75 = \boxed{j\,15.75}$

*The value of admittance can be read from the intersection of OP_4 with $r=0$ circle & x-circles (diametrically across of OP_3). This is $x \simeq 4.7 \Rightarrow Y_i = -\frac{j\,4.7}{75} = \boxed{-j\,0.063}$


IMPEDANCE OR ADMITTANCE COORDINATES
RESISTANCE COMPONENT (R/Zo), OR CONDUCTANCE COMPONENT (G/Yo)
INDUCTIVE REACTANCE COMPONENT (+jX/Zo), OR CAPACITIVE SUSCEPTANCE COMPONENT (+jB/Yo)
CAPACITIVE REACTANCE COMPONENT
WAVELENGTHS TOWARD GENERATOR
WAVELENGTHS TOWARD LOAD
ANGLE OF REFLECTION COEFFICIENT IN DEGREES
RADIALLY SCALED PARAMETERS
TOWARD GENERATOR
TOWARD LOAD
CENTER

Q: a) Express $V(z)$ & $I(z)$ in terms of the load voltage (V_L) & load current (I_L) in exponential form & in hyperbolic form

b) Express $V(z)$ & $I(z)$ in terms of the Input voltage (V_i) & Input current (I_i) at the input end in exponential form & hyperbolic form.

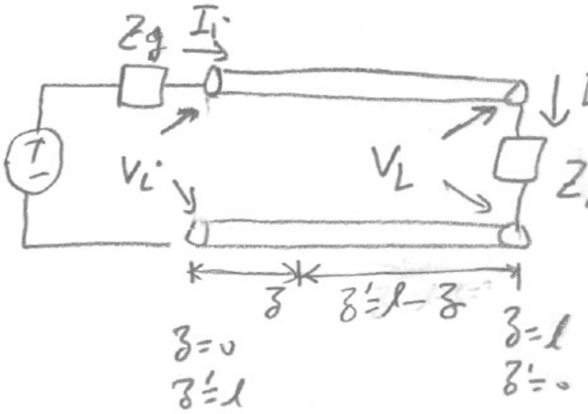
Sol:

a) The point of this exercise is to show that starting with

(1) $\boxed{V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}}$ & (2) $I(z) = I_0^+ e^{-\gamma z} + I_0^- e^{+\gamma z}$

We express (1) & (2) in terms of V_L & I_L or V_i & I_i

Note that since $\dfrac{V_0^+}{I_0^+} = -\dfrac{V_0^-}{I_0^-} = Z_0$



(2) $\Rightarrow$ $\boxed{I(z) = \dfrac{V_0^+}{Z_0} e^{-\gamma z} - \dfrac{V_0^-}{Z_0} e^{\gamma z}}$

* The part (a) was done in class and we saw that

$$\boxed{\begin{aligned} V(z') &= \frac{I_L}{2}\left[(Z_L+Z_0) e^{\gamma z'} + (Z_L - Z_0) e^{-\gamma z'}\right] \\ I(z') &= \frac{I_L}{2Z_0}\left[(Z_L+Z_0) e^{\gamma z'} - (Z_L - Z_0) e^{-\gamma z'}\right] \end{aligned}}$$

OR

$$\boxed{\begin{aligned} V(z') &= I_L (Z_L \cosh \gamma z' + Z_0 \sinh \gamma z') \\ I(z') &= \frac{I_L}{Z_0} (Z_L \sinh \gamma z' + Z_0 \cosh \gamma z') \end{aligned}}$$

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b)

To expren the (1) $V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$ &

(2) $I(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z} = \frac{V_0^+}{Z_0} e^{-\gamma z} - \frac{V_0^-}{Z_0} e^{\gamma z}$

in terms of input voltag V_i & input current I_i, we note the fact the (3) $V_i = V(z=0)$ & (4) $I_i = I(z=0)$ then

(5) $V_i = V_0^+ + V_0^-$

(6) $I_i = \frac{V_0^+}{Z_0} - \frac{V_0^-}{Z_0}$

Now (5) & (6) are two equations & 2 unknowns (V_0^+ & V_0^-) that can be solved for V_0^+ & V_0^-

(7) $V_i = V_0^+ + V_0^-$

(8) $Z_0 I_i = V_0^+ - V_0^-$ $\Rightarrow$ (9) $\frac{Z_0 I_i + V_i}{2} = V_0^+$ &

(10) $\frac{V_i - Z_0 I_i}{2} = V_0^-$

sub (9) & (10) in (1) & (2)

(11) $V(z) = \frac{1}{2}\left[Z_0 I_i + V_i\right] e^{-\gamma z} + \frac{1}{2}\left[V_i - Z_0 I_i\right] e^{\gamma z}$

(12) $\boxed{V(z) = -\frac{1}{2} Z_0 I_i \left(e^{\gamma z} - e^{-\gamma z}\right) + \frac{1}{2} V_i \left(e^{\gamma z} + e^{-\gamma z}\right)}$

(13) $I(z) = \frac{1}{Z_0} e^{-\gamma z}\left[\frac{Z_0 I_i + V_i}{2}\right] - \frac{e^{\gamma z}}{Z_0}\left[\frac{V_i - Z_0 I_i}{2}\right]$

(14) $\boxed{I(z) = \frac{I_i}{2}\left[e^{\gamma z} + e^{-\gamma z}\right] - \frac{V_i}{2Z_0}\left[e^{\gamma z} - e^{-\gamma z}\right]}$

from (12-P9) & (14-P9) it is clear that we can write

① $$\boxed{V(z) = V_i \cosh(\kappa z) - Z_0 I_i \sinh(\kappa z)}$$ &

② $$\boxed{I(z) = I_i \cosh(\kappa z) - \frac{V_i}{Z_0} \sinh(\kappa z)}$$

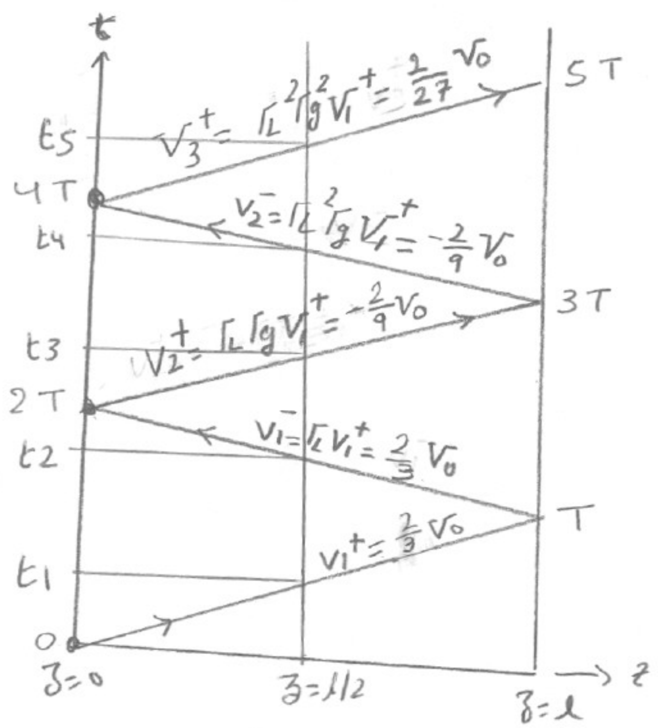
Q: A d-c voltage V_0 is applied at $t=0$ to the input terminals of an open-circuited air-dielectric line of length l through a series resistance equal to $R_0/2$, where R_0 is the characteristic resistance of the line a) Draw the voltage reflection diagram b) Sketch $V(z=0,t)$ c) sketch $V(l/2,t)$

Sol:

a) $\Gamma_L = 1$, $\Gamma_g = \dfrac{\frac{R_0}{2} - R_0}{\frac{R_0}{2} + R_0} = -\dfrac{1}{3}$, $V_1^+ = V_0 \dfrac{R_0}{R_0 + \frac{R_0}{2}} = \dfrac{2}{3} V_0 = 0.67\, V_0$

$$V_L = V_1^+ \frac{1+\Gamma_L}{1-\Gamma_L \Gamma_g} = V_1^+ \frac{1+1}{1+\frac{1}{3}} = \frac{3}{2} V_1^+ = \frac{3}{2}\,\frac{2}{3} V_0 = V_0$$

as expected for an open ckt

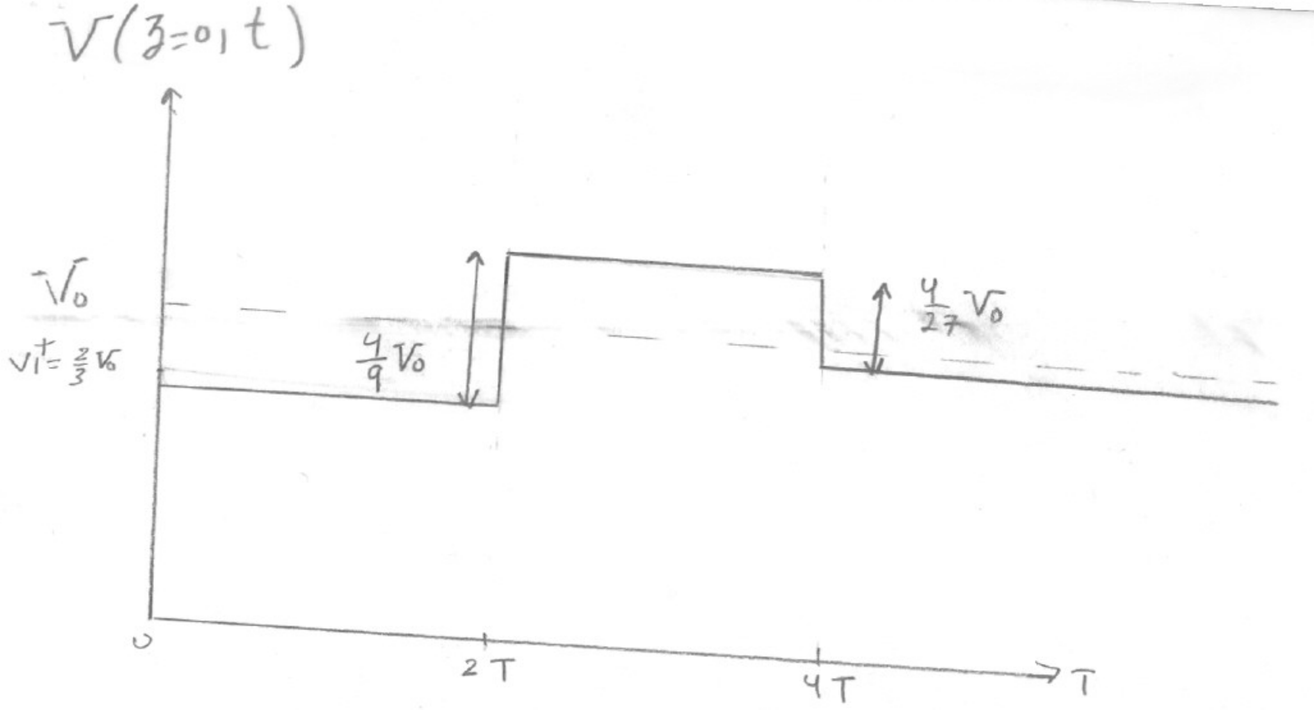


* To find $V(z=0,t)$ draw a vertical line at $z=0$. This intersects the directed lines at $t=0$, $t=2T$, $t=4T$, ..

* The points 0, 2T, 4T, 6T are important since new voltages appear on the line

b)

Time interval	voltage on the line	Discontinuity
$0 \le t < 2T$	$V_1^+ = \frac{2}{3} V_0$	$V_1^+ = \frac{2}{3} V_0$ at $t=0$
$2T \le t < 4T$	$V_1^+ + V_1^- + V_2^+ = V_1^+(1+\Gamma_L+\Gamma_L\Gamma_g)$ $= \frac{10}{9} V_0$	$V_1^+(\Gamma_L + \Gamma_L\Gamma_g) = \frac{4}{9} V_0$ at $2T$
$4T \le t < 6T$	$V_1^+ + V_1^- + V_2^+ + V_2^- + V_3^+ = V_1^+(1+\Gamma_L+\Gamma_L\Gamma_g+\Gamma_L^2\Gamma_g+\Gamma_L^2\Gamma_g^2)$ $= \frac{26}{27} V_0$	$V_1^+(\Gamma_L^2\Gamma_g + \Gamma_L^2\Gamma_g^2) = -\frac{4}{27} V_0$ at $4T$



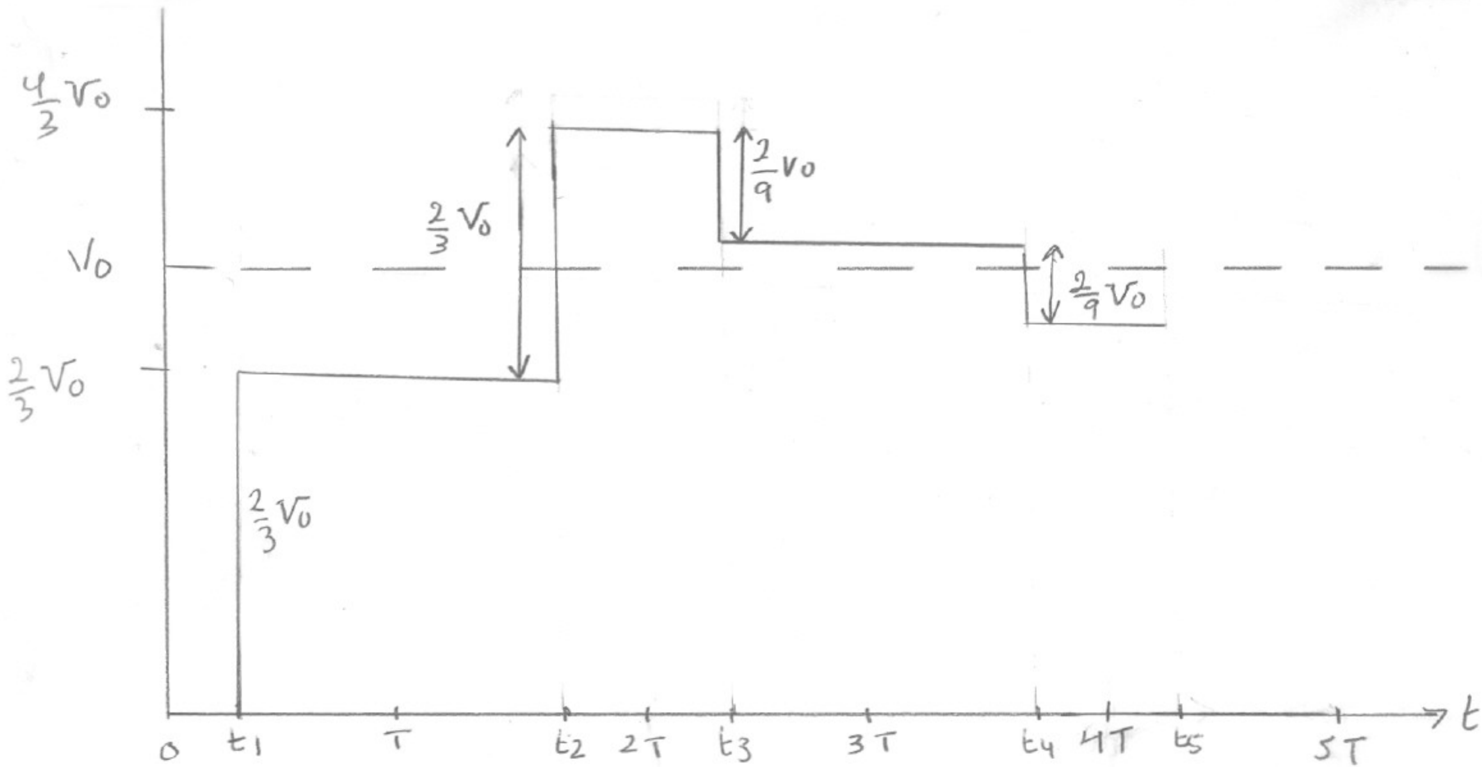
* Note that Voltage goes above V_0 for some time & drops below V_0 at other time interval, while it is approaching the value of $V_L = V_0$

c) for $V(z=l/2, t)$ we draw the line at $z=l/2$ (shown in red)

The important time points are $t_1, t_2, t_3, t_4, \ldots$

Time interval	voltage on the line	Discontinuity
$0 < t < t_1$	0	0
$t_1 \leq t < t_2$	$V_1^+ = \frac{2}{3}V_0$	$V_1^+ = \frac{2}{3}V_0$ at t_1
$t_2 \leq t < t_3$	$V_1^+ + V_1^- = V_1^+(1+\Gamma_L) = \frac{4}{3}V_0$	$V_1^+\Gamma_L = \frac{2}{3}V_0$ at t_2
$t_3 \leq t < t_4$	$V_1^+ + V_1^- + V_2^+ = V_1^+(1+\Gamma_L+\Gamma_L\Gamma_g)$ $= \frac{10}{9}V_0$	$V_1^+\Gamma_L\Gamma_g = -\frac{2}{9}V_0$ at t_3
$t_4 \leq t < t_5$	$V_1^+ + V_1^- + V_2^+ + V_2^- = V_1^+(1+\Gamma_L+\Gamma_L\Gamma_g+\Gamma_L^2\Gamma_g)$ $= \frac{8}{9}V_0$	$V_1^+\Gamma_L^2\Gamma_g = -\frac{2}{9}V_0$ at t_4

Note $t_1 = \frac{l/2}{U}$, $t_2 = 2T - t_1$, $t_3 = 2T + t_1$, $t_4 = 4T - t_1$, $t_5 = 4T + t_1$



$\frac{4}{3}V_0$
V_0
$\frac{2}{3}V_0$
$\frac{2}{3}V_0$
$\frac{2}{9}V_0$
$\frac{2}{9}V_0$
$\frac{2}{3}V_0$
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t_1
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