Answer the following questions:

Q1:

a) What is the defining equation for a lossy transmission line (TL) characteristic impedance. (1 pt) $Z_0 = \frac{V_0^+}{I_0^+} = -\frac{V_0^-}{I_0^-}$

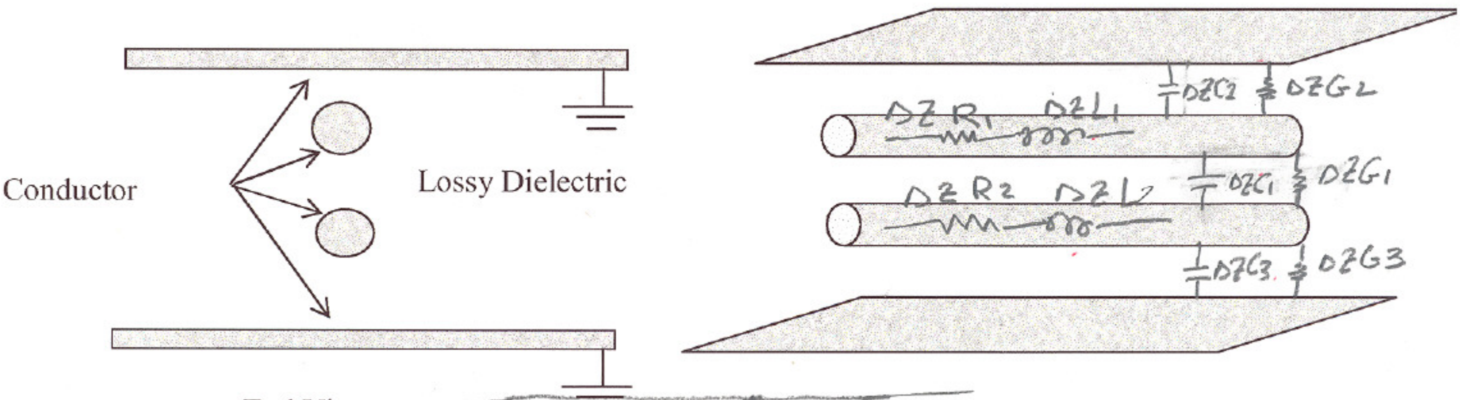
b) Give an expression for a lossy transmission line characteristic impedance in terms of the line capacitance, inductance, etc. (1 Pt) $Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C'}}$

c) Write the voltage wave equation for a lossy transmission line . (Make sure you define your variables) (1 pt) $\frac{d^2 V(z)}{dz^2} = \gamma^2 V(z)$ where $\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$

d) Write the solution to the equation you found in part (c). (Make sure you define your variables) (1 pt) $V = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}$ V_0^+ & V_0^- are constants

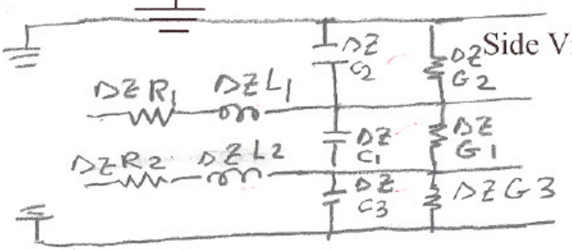
e) Define the voltage reflection coefficient at the load, and explain what do we mean by a matched line. (1 pt) $\Gamma(z=l) = \frac{V_0^- e^{\gamma l}}{V_0^+ e^{-\gamma l}} = \frac{Z_L - Z_0}{Z_L + Z_0}$; for matched line $Z_L = Z_0 \Rightarrow$ $\Gamma(z=l) = \Gamma_L = 0$

f) Figures below show a shielded two-wire transmission line. On the figure to the right draw a distributed parameter model for this two wire transmission line. (Make sure you indicated what each element of your modal represents) (5 pts)



End View

Side View



Q2:

$V_g = 10\angle 0$

$R_o = 50 \ [\Omega]$

$l = 50 \ [m]$

$\omega = 2\pi \times 10^8 \ [Hz]$

$Z_g = R_g = 25 \ [\Omega]$

$R_L = 50 \ [\Omega]$

$V_p = 1 \times 10^8 \ [m/s]$

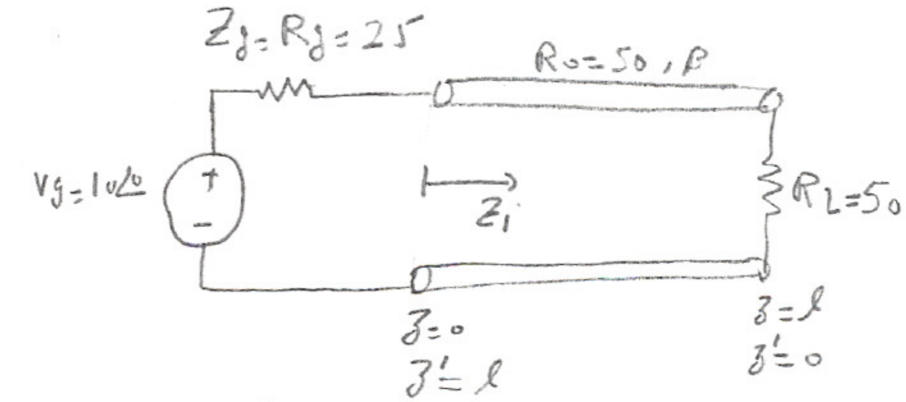
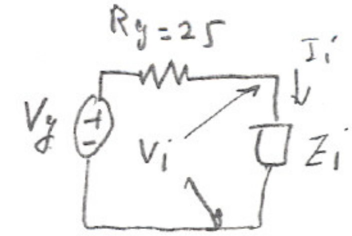


Fig 1

$z' + z = l$

a) We find $V(z)$ & $I(z)$ in phasor form & from them obtain the instantenous $v(z,t)$ & $i(z,t)$.

As far as the input to the line is concerned the Fig. 1 can be replaced with



where Z_i is the impedance at the input to the line

① $Z_i = R_o \dfrac{R_L + jR_o \tan\beta l}{R_o + jR_L \tan\beta l}$ for $R_L = R_o \Rightarrow$

② $Z_i = R_o$

③ then $V_i = V_g \dfrac{Z_i}{Z_i + R_g} = V_g \dfrac{R_o}{R_o + R_g} = 10 \dfrac{50}{50+25} = \dfrac{500}{75} = \dfrac{20}{3} = \boxed{6.667 \ [V]}$

④ $I_i = \dfrac{V_i}{Z_i} = \dfrac{V_i}{R_o} = \dfrac{V_g}{R_o + R_g} = \dfrac{10}{50+25} = \dfrac{10}{75} = \dfrac{2}{15} = \boxed{0.1333 \ [A]}$

× since line is matched we can write

(5) $V = V_i e^{-j\beta z}$ &

(6) $I = I_i e^{-j\beta z}$ (7)

but what is β

$$\boxed{\beta = \frac{\omega}{V_p} = \frac{2\pi \times 10^8}{1 \times 10^8} = 2\pi}$$

If we want we can calculate $\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{2\pi} = 1$

Then $V(z) = V_i e^{-j\beta z} = \boxed{6.667 e^{-j2\pi z}}$ &

$I(z) = I_i e^{-j\beta z} = \boxed{0.1333 e^{-j2\pi z}}$

The instantaneous fields are given by

$$v(z,t) = Re[V(z) e^{j\omega t}] = Re[6.667 e^{-j2\pi z} e^{j\omega t}] \Rightarrow$$

$$\boxed{v(z,t) = 6.667 \cos(\omega t - 2\pi z) = 6.667 \cos(2\pi \times 10^8 t - 2\pi z)} \; [V]$$

$$i(z,t) = Re[I(z) e^{j\omega t}] = Re[0.1333 e^{-j2\pi z} e^{j\omega t}] \Rightarrow$$

$$\boxed{i(z,t) = 0.1333 \cos(\omega t - 2\pi z) = 0.1333 \cos(2\pi \times 10^8 t - 2\pi z)} \; [Amp]$$

b) At load $z = l = 5$

$$\boxed{v_L = 6.667 \cos(2\pi \times 10^8 t - 10\pi) = 6.667 \cos(2\pi \times 10^8 t)} \; [V]$$

$$\boxed{i_L = 0.1333 \cos(2\pi \times 10^8 t - 10\pi) = 0.1333 \cos(2\pi \times 10^8 t)} \; [Amp]$$

c) $(P_{ave})_{Load} = (P_{ave})_{in}$ since matched line & lossless $\Rightarrow$

$$(P_{ave})_{load} = \frac{1}{2} Re[V_i I_i^*] = \frac{1}{2} Re[6.667 \times 0.1333] = 0.444 \; [watt]$$

Q3:

Sol: ① $V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{+j\beta z}$

② $I(z) = I_0^+ e^{-j\beta z} + I_0^- e^{j\beta z}$ ③

We first recognize the fact that since $\frac{V_0^+}{I_0^+} = \frac{-V_0^-}{I_0^-} = Z_0$ (1) & (2) can be written as -

④ $V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{+j\beta z}$

⑤ $I(z) = \frac{V_0^+}{Z_0} e^{-j\beta z} - \frac{V_0^-}{Z_0} e^{j\beta z}$

Also note that the line is lossless.

* At $z=0$ (B.C) we have $\boxed{V_i = V_0^+ + V_0^-}$ ⑥ &

$I_i = \frac{V_0^+}{Z_0} - \frac{V_0^-}{Z_0}$ ⑦ $\Rightarrow \boxed{I_i Z_0 = V_0^+ - V_0^-}$ ⑧

(6) & (8) are two unknown (V_0^+ & V_0^-) & two equations that can be solved $\Rightarrow$

⑨ $\boxed{V_0^+ = \frac{V_i + I_i Z_0}{2}}$ & $\boxed{V_0^- = \frac{V_i - I_i Z_0}{2}}$ ⑩

sub (9) & (10) in (4) & (5) & we have

⑪ $V(z) = \left(\frac{V_i + I_i Z_0}{2}\right) e^{-j\beta z} + \left(\frac{V_i - I_i Z_0}{2}\right) e^{j\beta z}$

⑫ $I(z) = \left(\frac{V_i + I_i Z_0}{2 Z_0}\right) e^{-j\beta z} - \left(\frac{V_i - I_i Z_0}{2 Z_0}\right) e^{j\beta z}$

or equally well

(13) $$\boxed{V(z) = V_i \frac{e^{j\beta z} + e^{-j\beta z}}{2} - I_i Z_0 \frac{e^{j\beta z} - e^{-j\beta z}}{2}}$$

(14) $$\boxed{I(z) = I_i \frac{e^{j\beta z} + e^{-j\beta z}}{2} - \frac{V_i}{Z_0} \frac{e^{j\beta z} - e^{-j\beta z}}{2}}$$

b) Above can also be written as:-

(15) $$\boxed{V(z) = V_i \cos(\beta z) - j I_i Z_0 \sin(\beta z)}$$

(16) $$\boxed{I(z) = I_i \cos(\beta z) - j \frac{V_i}{Z_0} \sin(\beta z)}$$

Q 4:

$l = 0.5\ [m]$ $\qquad f_1 = 100 \times 10^6\ [Hz]$

$R_0 = 50\ [\Omega]$ $\qquad f_2 = 200 \times 10^6\ [Hz]$

$V_P = 3 \times 10^8$

we know that $Z_i = R_0 \dfrac{R_L + jR_0 \tan(\beta l)}{R_0 + jR_L \tan(\beta l)}$ $\qquad \beta = \omega$

If $R_L = 0 \Rightarrow Z_i = jR_0 \tan(\beta l)$

a) $\beta_1 = \dfrac{\omega_1}{V_P} = \dfrac{2\pi \times 100 \times 10^6}{3 \times 10^8} = 2\pi/3$

$Z_1 = j50 \tan\left(\dfrac{2\pi}{3} \times 0.5\right) = \boxed{j\,86.6025}$

This looks like the impedance of an inductor for which

$Z = j\omega L \Rightarrow \quad L = \dfrac{86.6025}{2\pi \times 100 \times 10^6} = \boxed{1.378 \times 10^{-7}\ [H]}$

b) $\beta_2 = \dfrac{\omega_2}{V_P} = \dfrac{2\pi \times 200 \times 10^6}{3 \times 10^8} = \dfrac{4\pi}{3}$

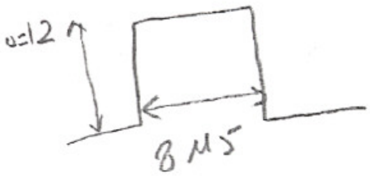
$Z_i = j50 \tan\left(\dfrac{4\pi}{3} \times 0.5\right) = \boxed{-j\,86.6025\ [\Omega]}$

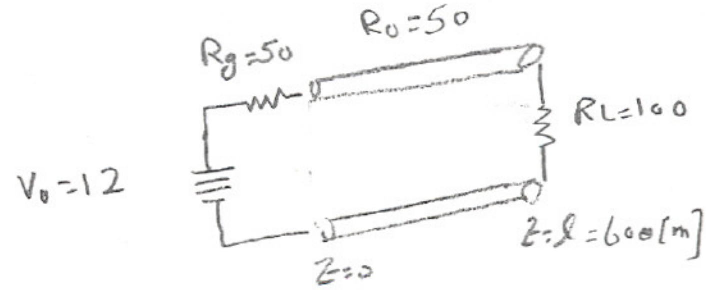
It looks like the impedance of a capacitor for which

$Z = \dfrac{-j}{\omega C} \Rightarrow \quad \dfrac{1}{\omega C} = 86.6025 \Rightarrow C = \dfrac{1}{2\pi \times 200 \times 10^6 \times 86.6025} \Rightarrow$

$\boxed{C \approx 9.1888 \times 10^{-12}\ [F]}$

Q5:





* Time to go from $z=0$ to $z=L$

$$T=\frac{L}{V_p}=\frac{600}{2\times10^{8}}=3\times10^{-6}=3\,[\mu s]$$

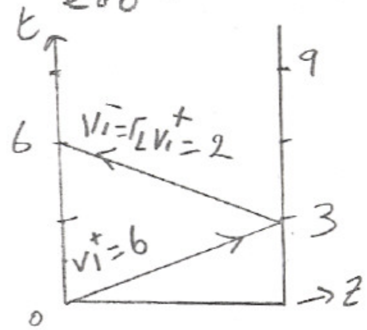
$$V_p=2\times10^{8}\ m/s$$

$$V_1^+=12\times\frac{50}{50+50}=6\,[V]$$

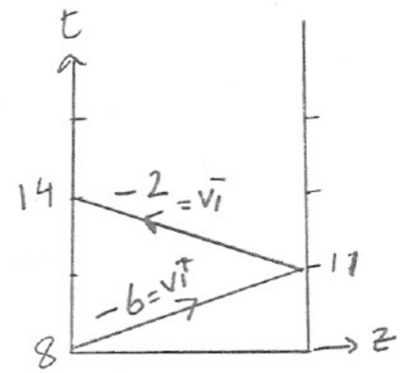
$$\Gamma_L=\frac{100-50}{100+50}=\frac{50}{150}=1/3$$

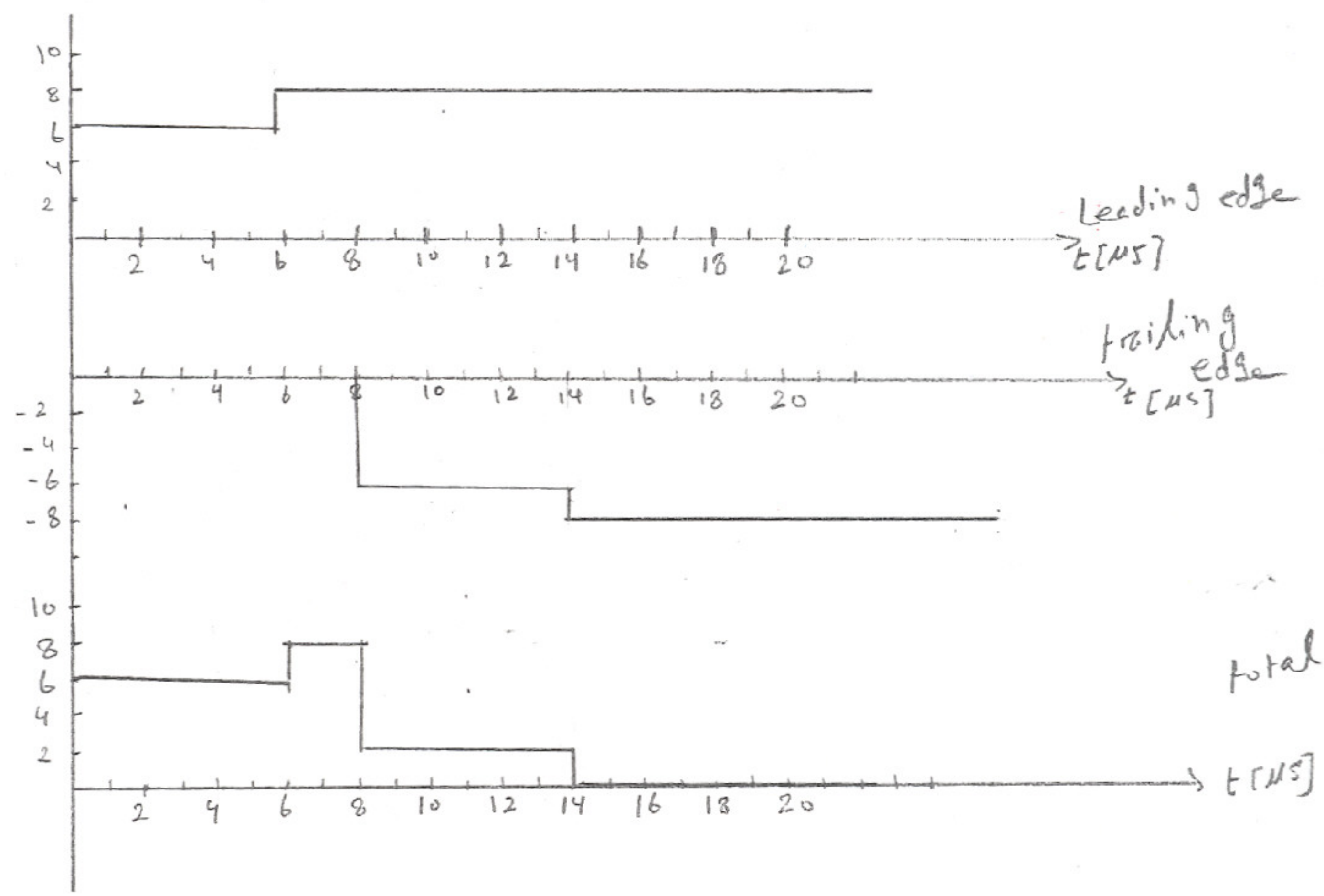
$$\Gamma_g=\frac{R_g-R_0}{R_g+R_0}=0$$

Reflection diagram for leading edge



Reflection diagram for the tail





$Z_0 + Z_g \qquad (1 - \Gamma_g \Gamma e$

Q.6:

a) $V_i = \frac{Z_0}{Z_0 + Z_g} V_g = \frac{300}{300+300} 60\angle 0^\circ = \boxed{30\angle 0^\circ \ V}$

$-\frac{8\pi}{5} = -5.03$

b) $V_L = V_i e^{-j\beta l} = 30\angle 0^\circ e^{-j\frac{4\pi}{5}\cdot 2} = \boxed{30\angle -\frac{8\pi}{5} \text{ or } 30\angle -288^\circ}$

$l = 2m, \ \beta = \frac{\omega}{v_p} = \frac{2\pi\, 100\times 10^6}{2.5\times 10^8} = \frac{2\pi}{2.5} = \frac{4\pi}{5} \ m^{-1}$

c) $\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{300-300}{300+300} = 0, \quad S = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+0}{1-0} = \boxed{1}$

d) $P_{ave} = \frac{1}{2}\frac{|V_L|^2}{R_L} = \frac{1}{2}\frac{|30\angle -\frac{8\pi}{5}|^2}{300} = \frac{30^2}{2\cdot 300} = \boxed{1.5\,W}$

e) $Z_L = \frac{1}{\frac{1}{300}+\frac{1}{300}} = 150\,\Omega, \quad \Gamma = \frac{150-300}{150+300} = \frac{-1}{3}, \quad S = \frac{1+\frac{1}{3}}{1-\frac{1}{3}} = \boxed{2}$

f) $z = l, \ z' = l - z = 0, \ \Gamma_g = \frac{Z_g - Z_0}{Z_g + Z_0} = 0 \quad V(z') = \frac{Z_0 V_g}{Z_0 + Z_g} e^{-j\beta z}\left(\frac{1+\Gamma e^{-2j\beta z'}}{1-\Gamma_g \Gamma e^{-2j\beta l}}\right) \Rightarrow$

$V(0) = \frac{300}{300+300} 60 e^{-j\beta l}\left[\frac{1-\frac{1}{3}e^{-j2\beta\cdot 0}}{1-0}\right] = 30 e^{-j\frac{8\pi}{5}}\left(1-\frac{1}{3}\right) = 20\angle -\frac{8\pi}{5}$

$P_{ave} = \frac{1}{2}\frac{|V(0)|^2}{R_L} = \frac{1}{2}\frac{|20\angle -\frac{8\pi}{5}|^2}{150} = \frac{20^2}{2\cdot 150} = \boxed{1.33\,W}$