1 (a):

* for time harmonic fields: $\vec{\mathcal{E}} = \mathrm{Re}\left[\vec{E}\, e^{j\omega t}\right] \Rightarrow$

$$\hat{a_y} E_0 \sin\left(\frac{\pi}{a} x\right) \cos(\omega t - \beta_z z) = \mathrm{Re}\left[E_0 \sin\left(\frac{\pi}{a} x\right) e^{-j\beta_z z} e^{j\omega t}\right]\hat{a_y}$$

$$\Rightarrow \boxed{\vec{E} = E_0 \sin\left(\frac{\pi}{a} x\right) e^{-j\beta_z z}\, \hat{a_y} = E_y\, \hat{a_y}}$$

$$\nabla \times \vec{E} = -j\omega \vec{B} \Rightarrow \vec{B} = \frac{j}{\omega} \nabla \times \vec{E} = \frac{j}{\omega} \begin{vmatrix} \hat{a_x} & \hat{a_y} & \hat{a_z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & E_y & 0 \end{vmatrix}$$

$$= \frac{j}{\omega}\left[-\hat{a_x} \frac{\partial}{\partial z} E_y + \hat{a_z} \frac{\partial}{\partial x} E_y\right]$$

$$= \frac{j}{\omega}\left[-\hat{a_x} E_0 \sin\left(\frac{\pi}{a} x\right)(-j\beta_z)\, e^{-j\beta_z z} + \hat{a_z} E_0 \frac{\pi}{a} \cos\left(\frac{\pi}{a} x\right) e^{-j\beta_z z}\right]$$

$$= -\frac{\beta_z}{\omega} E_0 \sin\left(\frac{\pi}{a} x\right) e^{-j\beta_z z}\, \hat{a_x} + \frac{j}{\omega} E_0 \frac{\pi}{a} \cos\left(\frac{\pi}{a} x\right) e^{-j\beta_z z}\, \hat{a_z}$$

since $\vec{H} = \dfrac{\vec{B}}{\mu_0}$ then

$$\vec{H} = -\frac{\beta_z}{\omega\mu_0} E_0 \sin\left(\frac{\pi}{a} x\right) e^{-j\beta_z z}\, \hat{a_x} + \frac{j E_0}{\omega\mu_0}\left(\frac{\pi}{a}\right) \cos\left(\frac{\pi}{a} x\right) e^{-j\beta_z z}\, \hat{a_z} \Rightarrow$$

$$\vec{H} = H_x \hat{a_x} + H_z \hat{a_z}$$

finally

$$\boxed{\vec{\mathcal{H}} = \mathrm{Re}\left[\vec{H} e^{j\omega t}\right] = -\frac{\beta_z}{\omega\mu_0} E_0 \sin\left(\frac{\pi}{a} x\right) \cos(\omega t - \beta_z z)\, \hat{a_x} + \frac{E_0}{\omega\mu_0}\left(\frac{\pi}{a}\right) \cos\left(\frac{\pi}{a} x\right) \cos\left(\omega t - \beta_z z + \frac{\pi}{2}\right)\hat{a_z}}$$

OR

$$\boxed{\vec{\mathcal{H}} = -\frac{\beta_z}{\omega\mu_0} E_0 \sin\left(\frac{\pi}{a} x\right) \cos(\omega t - \beta_z z)\, \hat{a_x} - \frac{E_0}{\omega\mu_0}\left(\frac{\pi}{a}\right) \cos\left(\frac{\pi}{a} x\right) \sin(\omega t - \beta_z z)\, \hat{a_z}}$$

1:(b)

From Ampere's law we have $\underline{\underline{2}}$

$$\nabla \times \vec{H} = \underbrace{\vec{J}}_{0} + j\omega\varepsilon_0 \vec{E} \Rightarrow \nabla \times \vec{H} = j\omega\varepsilon_0 \vec{E} \text{ here}$$

$$\vec{H} = H_x \hat{a_x} + H_z \hat{a_z} \text{ then } \nabla \times \vec{H} = \begin{vmatrix} \hat{a_x} & \hat{a_y} & \hat{a_z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ H_x & 0 & H_z \end{vmatrix} \Rightarrow$$

$$\nabla \times \vec{H} = \hat{a_x} \underbrace{\frac{\partial}{\partial y} H_z}_{0} - \hat{a_y}\left(\frac{\partial}{\partial x} H_z - \frac{\partial}{\partial z} H_x\right)$$

$$- \hat{a_z}\left(\underbrace{\frac{\partial}{\partial y} H_x}_{0}\right) = \hat{a_y}\left(\frac{\partial}{\partial z} H_x - \frac{\partial}{\partial x} H_z\right) \text{ \& since}$$

$$\nabla \times \vec{H} = j\omega\vec{\varepsilon_0 E} \Rightarrow \hat{a_y}\left(\frac{\partial}{\partial z} H_x - \frac{\partial}{\partial x} H_z\right) = j\omega\varepsilon_0 E_y \hat{a_y} \Rightarrow$$

$$\frac{-\beta_z}{\omega\mu_0} \cancel{E_0} \cancel{\sin\left(\frac{\pi}{a}x\right)} (-j\beta_z) \cancel{e^{-j\beta_z z}} + \frac{j\cancel{E_0}}{\omega\mu_0}\left(\frac{\pi}{a}\right)^2 \cancel{\sin\left(\frac{\pi}{a}x\right)} \cancel{e^{-j\beta_z z}} =$$

$$j\omega\varepsilon_0 \cancel{E_0} \cancel{\sin\left(\frac{\pi}{a}x\right)} \cancel{e^{-j\beta_z z}} \Rightarrow$$

$$\frac{\cancel{j}\beta_z^2}{\omega\mu_0} + \frac{\cancel{j}}{\omega\mu_0}\left(\frac{\pi}{a}\right)^2 = \cancel{j}\omega\varepsilon_0 \Rightarrow$$

$$\beta_z^2 + \left(\frac{\pi}{a}\right)^2 = \omega^2\mu_0\varepsilon_0 \Rightarrow \boxed{\beta_z = \pm\sqrt{\omega^2\mu_0\varepsilon_0 - \left(\frac{\pi}{a}\right)^2}}$$

Problem **2**:

① $\nabla\Psi = \frac{\partial\Psi}{\partial x}\hat{a}_x + \frac{\partial\Psi}{\partial y}\hat{a}_y + \frac{\partial\Psi}{\partial z}\hat{a}_z$ & from Coordinate transformation we have

② $$\begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}$$

when $\vec{A} = A_x\hat{a}_x + A_y\hat{a}_y + A_z\hat{a}_z = A_\rho\hat{a}_\rho + A_\phi\hat{a}_\phi + A_z\hat{a}_z$ - using (2) the vector $\nabla\Psi$ given in Cartesian coordinate can be transformed to to cylindrical coordinate according to

③ $$\begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \partial\Psi/\partial x \\ \partial\Psi/\partial y \\ \partial\Psi/\partial z \end{bmatrix} = \begin{bmatrix} \nabla_\rho \\ \nabla_\phi \\ \nabla_z \end{bmatrix} \Rightarrow$$

) $\nabla_\rho = \cos\phi\frac{\partial\Psi}{\partial x} + \sin\phi\frac{\partial\Psi}{\partial y}$

) $\nabla_\phi = -\sin\phi\frac{\partial\Psi}{\partial x} + \cos\phi\frac{\partial\Psi}{\partial y}$

) $\nabla_z = \frac{\partial\Psi}{\partial z}$

$\Rightarrow$ & ⑦ $\nabla\Psi(\rho,\phi,z) = \nabla_\rho\Psi(\rho,\phi,z)\hat{a}_\rho + \nabla_\phi\Psi(\rho,\phi,z)\hat{a}_\phi + \nabla_z\Psi(\rho,\phi,z)\hat{a}_z$

By chain rule

⑧ $\frac{\partial\Psi}{\partial x} = \frac{\partial\Psi}{\partial\rho}\frac{\partial\rho}{\partial x} + \frac{\partial\Psi}{\partial\phi}\frac{\partial\phi}{\partial x} + \frac{\partial\Psi}{\partial z}\cancel{\frac{\partial z}{\partial x}}^{\to 0}$

⑨ $\frac{\partial\Psi}{\partial y} = \frac{\partial\Psi}{\partial\rho}\frac{\partial\rho}{\partial y} + \frac{\partial\Psi}{\partial\phi}\frac{\partial\phi}{\partial y} + \frac{\partial\Psi}{\partial z}\cancel{\frac{\partial z}{\partial y}}^{\to 0}$

Therefor, we need to calculate $\frac{\partial \rho}{\partial x}, \frac{\partial \rho}{\partial y}, \frac{\partial \phi}{\partial x}, \frac{\partial \phi}{\partial y}$ $\underline{\underline{2}}$

* From transformation rule we have

(10) $\rho = (x^2+y^2)^{1/2}$ (12) $x = \rho \cos\phi$

(11) $\phi = \tan^{-1}(y/x)$ (13) $y = \rho \sin\phi$

(14)

then $$\frac{\partial \rho}{\partial x} = \frac{1}{\cancel{2}}(x^2+y^2)^{-1/2} \times \cancel{2}x = \frac{x}{(x^2+y^2)^{1/2}} = \frac{\rho\cos\phi}{\rho} = \boxed{\cos\phi}$$

(15) $$\frac{\partial \rho}{\partial y} = \frac{1}{\cancel{2}}(x^2+y^2)^{-1/2}\cancel{2}y = \frac{y}{(x^2+y^2)^{1/2}} = \frac{\rho\sin\phi}{\rho} = \boxed{\sin\phi}$$

(16) $$\frac{\partial \phi}{\partial x} = \frac{1}{[1+(y/x)^2]} \frac{-y}{x^2} = \frac{-y}{x^2+y^2} = \frac{-\rho\sin\phi}{\rho^2} = \boxed{\frac{-\sin\phi}{\rho}}$$

(17) $$\frac{\partial \phi}{\partial y} = \frac{1}{[1+(y/x)^2]} \frac{1}{x} = \frac{x}{(x^2+y^2)} = \frac{\rho\cos\phi}{\rho^{\cancel{2}}} = \boxed{\frac{\cos\phi}{\rho}}$$

To summarize

(18) $$\boxed{\frac{\partial \rho}{\partial x} = \cos\phi}$$

(19) $$\boxed{\frac{\partial \rho}{\partial y} = \sin\phi}$$

(20) $$\boxed{\frac{\partial \phi}{\partial x} = -\frac{\sin\phi}{\rho}}$$

(21) $$\boxed{\frac{\partial \phi}{\partial y} = \frac{\cos\phi}{\rho}}$$

* Let us put everything together, i.e use Eqs (4-5), (8-9), and (18-21)

$$\nabla_\rho = \cos\phi \frac{\partial \psi}{\partial x} + \sin\phi \frac{\partial \psi}{\partial y} = \cos\phi\left[\frac{\partial \psi}{\partial \rho}\frac{\partial \rho}{\partial x} + \frac{\partial \psi}{\partial \phi}\frac{\partial \phi}{\partial x}\right] + \sin\phi\left[\frac{\partial \psi}{\partial \rho}\frac{\partial \rho}{\partial y} + \frac{\partial \psi}{\partial \phi}\frac{\partial \phi}{\partial y}\right]$$

$$= \cos\phi\left[\frac{\partial \psi}{\partial \rho}\cos\phi + \frac{\partial \psi}{\partial \phi}\left(-\frac{\sin\phi}{\rho}\right)\right] + \sin\phi\left[\frac{\partial \psi}{\partial \rho}\sin\phi + \frac{\partial \psi}{\partial \phi}\left(\frac{\cos\phi}{\rho}\right)\right]$$

$$= \cos^2\phi \frac{\partial \psi}{\partial \rho} - \cancel{\frac{\sin\phi\cos\phi}{\rho}}\frac{\partial \psi}{\partial \phi} + \sin^2\phi\frac{\partial \psi}{\partial \rho} + \cancel{\frac{\sin\phi\cos\phi}{\rho}}\frac{\partial \psi}{\partial \phi} = \boxed{\frac{\partial \psi}{\partial \rho}}$$

For ∇_ϕ we have

$$\nabla_\phi = -\sin\phi\frac{\partial \psi}{\partial x} + \cos\phi\frac{\partial \psi}{\partial y} = -\sin\phi\left[\frac{\partial \psi}{\partial \rho}\frac{\partial \rho}{\partial x} + \frac{\partial \psi}{\partial \phi}\frac{\partial \phi}{\partial x}\right] + \cos\phi\left[\frac{\partial \psi}{\partial \rho}\frac{\partial \rho}{\partial y} + \frac{\partial \psi}{\partial \phi}\frac{\partial \phi}{\partial y}\right]$$

$$= -\sin\phi\left[\frac{\partial \psi}{\partial \rho}\cos\phi + \frac{\partial \psi}{\partial \phi}\left(-\frac{\sin\phi}{\rho}\right)\right] + \cos\phi\left[\frac{\partial \psi}{\partial \rho}\sin\phi + \frac{\partial \psi}{\partial \phi}\frac{\cos\phi}{\rho}\right]$$

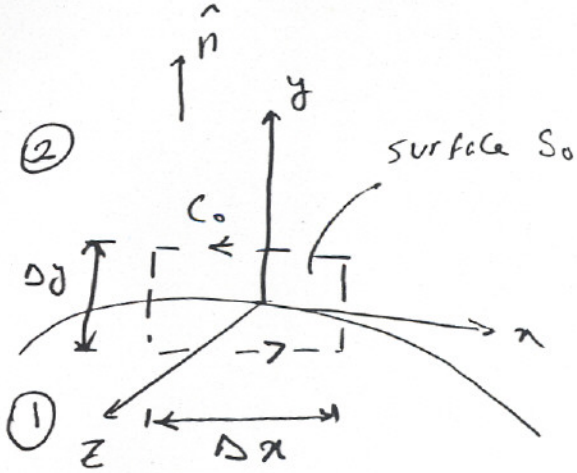
$$= \cancel{-\sin\phi\cos\phi}\frac{\partial \psi}{\partial \rho} + \frac{\sin^2\phi}{\rho}\frac{\partial \psi}{\partial \phi} + \cancel{\sin\phi\cos\phi}\frac{\partial \psi}{\partial \rho} + \frac{\cos^2\phi}{\rho}\frac{\partial \psi}{\partial \phi}$$

$= \boxed{\frac{1}{\rho}\frac{\partial \psi}{\partial \phi}}$ & we have seen that $\nabla_z = \frac{\partial}{\partial z}\psi$.

& Then from (7) we have

$$\boxed{\nabla\psi(\rho,\phi,z) = \frac{\partial \psi(\rho,\phi,z)}{\partial \rho}\hat{a}_\rho + \frac{1}{\rho}\frac{\partial \psi(\rho,\phi,z)}{\partial \phi}\hat{a}_\phi + \frac{\partial \psi(\rho,\phi,z)}{\partial z}\hat{a}_z}$$

3)



Amper law $\Rightarrow$

① $$\oint \vec{H}\cdot d\vec{l} = \iint \vec{J_i}\cdot d\vec{s} + \iint \sigma \vec{E}\cdot d\vec{s} + \frac{\partial}{\partial t}\iint \vec{D}\cdot d\vec{s} \Rightarrow$$

② $$\oint \vec{H}\cdot d\vec{l} = \iint \vec{J_{ic}}\cdot d\vec{s} + \frac{\partial}{\partial t}\iint \vec{D}\cdot d\vec{s}$$

where $\vec{J_{ic}} = \vec{J_i} + \vec{J_c} = \vec{J_i} + \sigma\vec{E}$ ③

The LHS of (2) $\Rightarrow$

$$\lim_{\Delta y \to 0} \oint \vec{H}\cdot d\vec{l} = \lim_{\Delta y} \left\{ \int \vec{H_1}\cdot d\vec{l} + \int \vec{H_2}\cdot d\vec{l} \right\} = \lim_{\Delta y \to 0} \left\{ \vec{H_1}\,\Delta x\cdot\hat{a}_x - \vec{H_2}\,\Delta x\cdot\hat{a}_x \right\}$$

$$= \boxed{(\vec{H_1} - \vec{H_2})\,\Delta x\cdot\hat{a}_x}$$ ④

The RHS of (2)

As before $$\lim_{\Delta y \to 0} \frac{\partial}{\partial t}\iint \vec{D}\cdot d\vec{s} = \lim_{\Delta y \to 0} \frac{\partial}{\partial t}\iint \vec{D}\cdot dx\,dy\,\hat{a}_z = \lim_{\Delta y \to 0} \frac{\partial}{\partial t}\left[\vec{D}\cdot\Delta x\,\Delta y\,\hat{a}_z\right]$$

$$= 0$$ ⑤

But

$$\lim_{\Delta y \to 0} \iint \vec{J_{ic}}\cdot d\vec{s} = \lim_{\Delta y \to 0} \iint \vec{J_{ic}}\cdot dx\,dy\,\hat{a}_z = \lim_{\Delta y \to 0} \left[\vec{J_{ic}}\,\Delta x\,\Delta y\right]\cdot\hat{a}_z$$

$$= \vec{J_s}\,\Delta x\cdot\hat{a}_z$$ ⑥

where

$$\boxed{\vec{J_s} = \lim_{\Delta y \to 0} \vec{J_{ic}}\,\Delta y}$$ ⑦

Putting the RHS & LHS together we have

(8) $(\vec{H}_1 - \vec{H}_2)\cancel{\Delta x} \cdot \hat{a}_x = \vec{J}_s \cancel{\Delta x} \cdot \hat{a}_z$

Note that $\hat{a}_x = \hat{a}_y \times \hat{a}_z$ then (8) $\Rightarrow$

(9) $(\vec{H}_1 - \vec{H}_2) \cdot (\hat{a}_y \times \hat{a}_z) = \vec{J}_s \cdot \hat{a}_z$

Recall $A \cdot (\vec{B} \times \vec{C}) = \vec{C} \cdot (\vec{A} \times \vec{B})$ (10)

then (9) $\Rightarrow$

(10) $\cancel{\hat{a}_z} \cdot [(\vec{H}_1 - \vec{H}_2) \times \hat{a}_y] = \cancel{\hat{a}_z} \cdot \vec{J}_s \Rightarrow$

(11) $\hat{a}_y \times (\vec{H}_2 - \vec{H}_1) = \vec{J}_s$ but from figure it is clear

that $\hat{a}_y = \hat{n} \Rightarrow$

$$\boxed{\hat{n} \times (\vec{H}_2 - \vec{H}_1) = \vec{J}_s}$$

problem (1)

Vector $\vec{M}_1$ is defined by

① $\nabla\cdot\vec{M}_1 = S$

② $\nabla\times\vec{M}_1 = \vec{C}$

and by its normal component $\vec{M}_{1n}$ over a boundary. We want to show that given the above, $\vec{M}_1$ is unique. To do so, let us suppose there is another vector $\vec{M}_2$ that also satisfies all the above conditions. We construct

③ $\vec{W} = \vec{M}_1 - \vec{M}_2$ & show that $W=0 \Rightarrow \vec{M}_1 = \vec{M}_2$

Note that ④ $\nabla\cdot\vec{W} = \nabla\cdot(\vec{M}_1 - \vec{M}_2) = \nabla\cdot\vec{M}_1 - \nabla\cdot\vec{M}_2 = S - S = 0$

& ⑤ $\nabla\times\vec{W} = \nabla\times(\vec{M}_1 - \vec{M}_2) = \nabla\times\vec{M}_1 - \nabla\times\vec{M}_2 = \vec{C} - \vec{C} = 0$

Since $\nabla\times\vec{W} = 0 \Rightarrow$ ⑦ $\vec{W} = -\nabla\phi$ use (7) in (4) we have

⑧ $\nabla\cdot W = \nabla\cdot(-\nabla\phi) = -\nabla\cdot\nabla\phi = -\nabla^2\phi = 0$: Laplace Eq

Green theorem tells us that

⑨ $$\iint_S U\nabla V\cdot d\vec{s} = \iiint_v U\nabla\cdot\nabla V\, dv + \iiint_v \nabla U\cdot\nabla V\, dv$$

In (9) let (10) $U = V = \phi$ then (9) $\Rightarrow$

⑪ $$\iint_S \phi\nabla\phi\cdot d\vec{s} = \iiint_v \phi\nabla\cdot\nabla\phi\, dv + \iiint_v \nabla\phi\cdot\nabla\phi\, dv$$

Problem 4)

(11) Can be written as

(12) $$\iiint_v \nabla\phi\cdot\nabla\phi\, dv = \iint_s \phi\nabla\phi\cdot\vec{ds} - \iiint_v \phi\nabla\cdot\nabla\phi\, dv$$

However (8) tell us that $\boxed{\nabla\cdot\nabla\phi = \nabla^2\phi = 0}$; (13)

Also note that $\iint_s \phi\nabla\phi\cdot\vec{ds} = \iint_s \phi(-\vec{W}\cdot ds) = -\iint \phi(\vec{W}\cdot\vec{ds})$,
From (7)

but what is $\iint_s \phi(\vec{W}\cdot\vec{ds})$? It is the integral of the normal components of $\vec{W}$ ($\vec{W}_n = \vec{M}_{1n} - \vec{M}_{2n}$); and we have supposed that the normal components of $\vec{M_1}$ & $\vec{M_2}$, i.e. $\vec{M}_{1n}$ & $\vec{M}_{2n}$ satisfy the same boundary conditions $\Rightarrow$

(14) $$\boxed{\begin{aligned}\iint_s \phi\nabla\phi\cdot\vec{ds} &= \iint_s \phi(\vec{W}\cdot\vec{ds}) \\ &= \iint \phi(M_{1n} - M_{2n})\, ds = 0\end{aligned}}$$

Let us use (13) & (14) in (12) $\Rightarrow$

(15) $\iiint_v \nabla\phi\cdot\nabla\phi\, dv = 0$ Now using (7), (15) can be written as (16)

$$\iiint_v -\vec{W}\cdot(-\vec{W})\, dv = 0 \Rightarrow \iiint \vec{W}\cdot\vec{W}\, dv = 0 \Rightarrow \iiint_v |\vec{W}|^2 dv = 0$$

since the norm of a vector $|\vec{W}|^2$ can not be negative, the only way we can satisfy (16) is $\vec{W} = 0 \Rightarrow \vec{M_1} = \vec{M_2}$

Problem #5)

$$\nabla^2 V = \frac{1}{R^2}\frac{\partial}{\partial R} R^2 \frac{\partial}{\partial R} V + \frac{1}{R^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial V}{\partial \theta}\right) + \frac{1}{R^2 \sin^2\theta}\frac{\partial^2 V}{\partial \phi^2}$$

for $V = \frac{1}{|\vec{R}|} = \frac{1}{R}$ then $\frac{\partial}{\partial \theta} \to 0 \quad \frac{\partial}{\partial \phi} \to 0 \Rightarrow$

$$\nabla^2 \frac{1}{R} = \frac{1}{R^2}\frac{\partial}{\partial R} R^2 \left(\frac{\partial}{\partial R}\frac{1}{R}\right) = \frac{1}{R^2}\frac{\partial}{\partial R}\cancel{R^2}\left[\frac{-1}{\cancel{R^2}}\right] = \frac{1}{R^2}\frac{\partial}{\partial R}(-1) = 0$$

For $R \neq 0 \Rightarrow$ $\boxed{\nabla^2 \frac{1}{R} = 0 \text{ for } R \neq 0}$

But for $R = 0$, $\frac{1}{R}$ & hence $\nabla^2 \frac{1}{R}$ is singular.

Note the following:

$$\nabla^2\left(\frac{1}{R}\right) = \nabla\cdot\left(\nabla \frac{1}{R}\right) \Rightarrow \iiint_v \nabla^2 \frac{1}{R}\, dv = \iiint_v \nabla\cdot\left(\nabla \frac{1}{R}\right) dv^3 \Rightarrow$$

$$\iiint_v \nabla^2 \frac{1}{R}\, dv^3 = \oiint_s \nabla \frac{1}{R}\cdot \vec{ds}$$ where s is the surface bounding volume v

using divergence theorem

* It also can be shown $\nabla\left(\frac{1}{R}\right) = \frac{-\hat{R}}{R^2}$ where $\hat{R}$ is the unit normal vector in direction $\vec{R}$

then

$$\iiint_v \nabla^2 \frac{1}{R}\, dv^3 = \oiint_s \nabla \frac{1}{R}\cdot \vec{ds} = \oiint_s -\frac{\hat{R}}{R^2}\cdot \vec{ds}$$

$$= \oiint -\frac{\hat{R}}{R^2}\cdot R^2 \sin\theta\, d\theta\, d\phi\, \hat{R}$$

where $\vec{ds} = R^2 \sin\theta\, d\theta\, d\phi\, \hat{R}$ for integration over the surface of the sphere

$$\iiint \nabla^2 \frac{1}{R}\, dv^3 = \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} -\sin\theta \, d\theta\, d\phi$$

(2) $$= -4\pi$$ hence

$$\boxed{\iiint \nabla^2 \frac{1}{R}\, dv^3 = -4\pi}$$

We now have shown that $\nabla^2 \frac{1}{R} = 0$ for all points with exception of $R=0$ & that its integral is -4π for the case of $R=0$. This is the property of Delta function for which $\delta(R)=0$ for $R \neq 0$ &

(3) $$\iiint_V \delta^3(R)\, dv = 1$$. In other words we can write (2)

as
$$-\frac{1}{4\pi}\iiint \nabla^2 \frac{1}{R}\, dv^3 = 1$$ & compare to

$$\iiint \delta^3(R)\, dv^3 = 1 \Rightarrow$$

$$-\frac{1}{4\pi}\nabla^2 \frac{1}{R} = \delta^3(R) \Rightarrow -\nabla^2 \frac{1}{R} = 4\pi\, \delta^3(R)$$

Problem 6)

(a) start with continuity Eq

① $\nabla \cdot \vec{J} + \frac{\partial \rho_{ev}}{\partial t} = 0$ where ② $\vec{J} = \sigma \vec{E}$ $\Rightarrow$

③ $\nabla \cdot \sigma \vec{E} + \frac{\partial \rho_{ev}}{\partial t} = 0 \Rightarrow$ ④ $\sigma \nabla \cdot \vec{E} + \frac{\partial \rho_{ev}}{\partial t} = 0$ but ⑤ $\nabla \cdot E = \rho_{ev}/\varepsilon \Rightarrow$

⑥ $$\boxed{\frac{\sigma}{\varepsilon} \rho_{ev} + \frac{\partial \rho_{ev}}{\partial t} = 0}$$

(b) The solution for ⑦ $\frac{\partial \rho_{ev}}{\partial t} = -\frac{\sigma}{\varepsilon} \rho_{ev}$ is given by

⑧ $\rho_{ev} = \rho_o e^{-\frac{\sigma}{\varepsilon} t}$ where $\rho_o \triangleq$ intial Charge distribution is a Constant

we define ⑨ $\tau = \varepsilon / \sigma$ then

⑩ $$\boxed{\rho_{ev} = \rho_o e^{-\frac{\sigma}{\varepsilon} t} = \rho_o e^{-t/\tau}}$$

(C) Eq. (10) states that if charge density ρ_o is placed inside the metal, its distribution will decrese with time in an exponential manner, i.e. the charges move to the metal surface & redistribute themselves such that $\vec{E}$ inside of metal is zero. The time constant for this redistribution is proportional to $\tau = \frac{\varepsilon}{\sigma}$, i.e. the more conductive a metal, the faster charge redistribution. For copper, charge density reaches e^{-1} (36.8% of original value) at $\tau = \frac{\varepsilon}{\sigma} = \frac{\varepsilon_o}{\sigma} = \frac{8.85 \times 10^{-12}}{5.8 \times 10^{7}}$

$\simeq 1.5 \times 10^{-19}$ [s]