Supplemental #2
ECE357 /ECE320
University of Toronto

1- The instantaneous electric field inside a conducting rectangular pipe (waveguide) is given by

$$\vec{E}(r,t) = \hat{a}_y\, E_0 \sin\left(\frac{\pi}{a}x\right)\cos(\omega t - \beta_z\, z)$$

where β_z is the waveguide's phase constant and *a* is the waveguide width (a constant). Assuming there are no sources within the free-space-filled pipe, determine

a) The corresponding instantaneous magnetic field components inside the conducting pipe.

b) The phase constant β_z.

2- If gradient of a scalar function ψ in rectangular coordinate system is given by

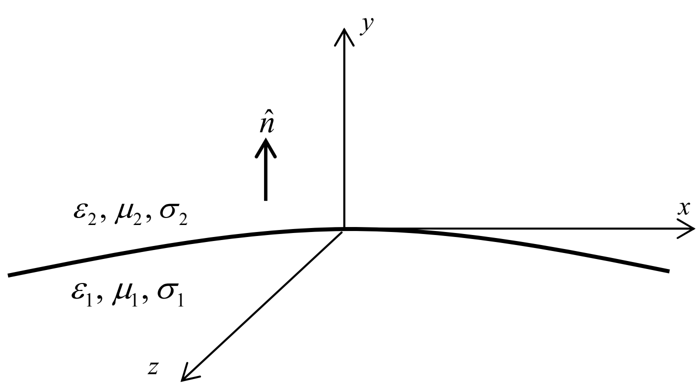
$\vec{\nabla}\,\psi = \frac{\partial \psi}{\partial x}\,\hat{a}_x + \frac{\partial \psi}{\partial y}\,\hat{a}_y + \frac{\partial \psi}{\partial z}\,\hat{a}_z$, using coordinate transformation and chain rule show that the gradient of ψ in cylindrical coordinate is given by

$$\vec{\nabla}\,\psi = \frac{\partial \psi}{\partial \rho}\,\hat{a}_\rho + \frac{1}{\rho}\frac{\partial \psi}{\partial \phi}\,\hat{a}_\phi + \frac{\partial \psi}{\partial z}\,\hat{a}_z\,.$$

3- In the class we showed that when there were no sources at the interface between two media and neither of the two media was a perfect conductor ($\sigma_1, \sigma_2 \neq \infty$) the boundary condition on the tangential magnetic field was given by $\hat{n}\times\left(\vec{H}_2 - \vec{H}_1\right) = 0$. Here, show that when $\vec{J}_i + \vec{J}_c = \vec{J}_{ic} \neq 0$, the B.C. is given by $\hat{n}\times\left(\vec{H}_2 - \vec{H}_1\right) = \vec{J}_s$, where,

$\vec{J}_s = \underset{\Delta y \to 0}{Lim}\,\vec{J}_{ic}\;\Delta y$.

Note: Used the geometry provided in figure below for your proof.



4- Prove first Helmholtz's theorem, i.e. if vector $\vec{M}_1$ is defined by its divergence ($\nabla \cdot \vec{M}_1 = s$) and its curl ($\nabla \times \vec{M}_1 = \vec{C}$) within a region, and its normal component $\vec{M}_{1n}$ over the boundary, then $\vec{M}_1$ is uniquely specified.
Note: Assume there is a vector $\vec{M}_2$ with its divergence and curl equal to s and $\vec{C}$ respectively, then show that $\vec{M}_1 = \vec{M}_2$.

5- Prove that $-\nabla^2 \dfrac{1}{R} = 4\pi\, \delta^3(R)$ where $R = |\vec{R}|$ is the position vector.

6- (a) For a linear, homogenous, and isotropic conductor (conductivity σ)show that electric charge density ρ_{ev} satisfies the following differential equation.

$$\frac{\partial \rho_{ev}}{\partial t} + \frac{\sigma}{\varepsilon}\rho_{ev} = 0$$

(b) Obtain the solution for the above differential equation.
(c) From your results in part (b) comment on what happens when a charge density is placed inside a good conductor. For copper, calculate the order of the magnitude for the time it takes for the charge density to reach 36.8% of its initial value.