Family Name:	Given name:
Student number	Signature

Faculty of Applied Science and Engineering

ECE320 Fields and Waves

First Test, October 19, 2006

Examiners – M. Mojahedi

Only Calculators approved by Registrar allowed
Answer the questions in the spaces provided or on the facing page
A complete paper consists of answers to all questions
For numerical answers specify units

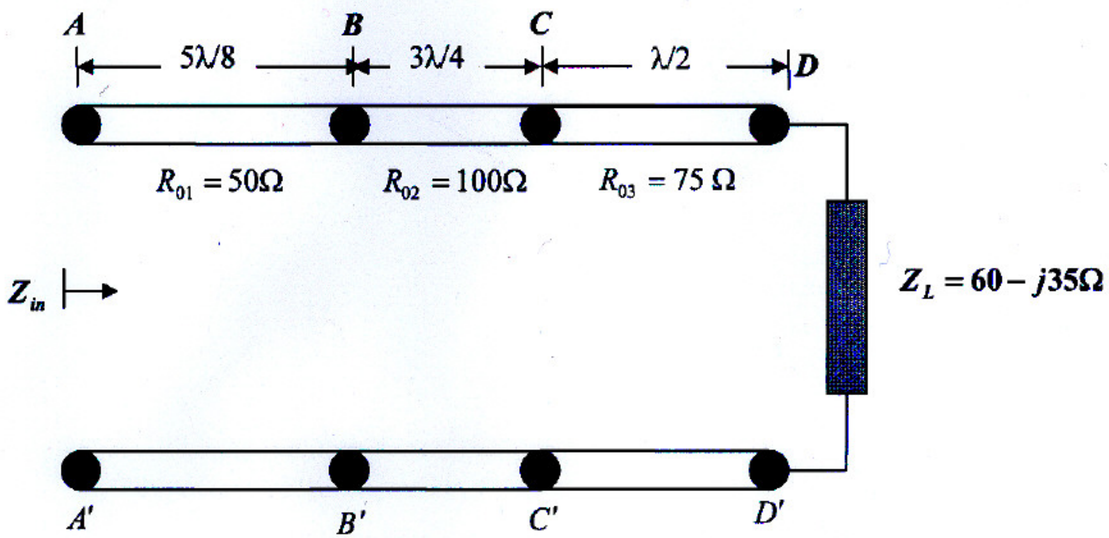
DO NOT REMOVE STAPLE

Do not write in these spaces

1	2	3	TOTAL

$\varepsilon_0 = 8.854 \times 10^{-12}\ [F/m], \quad \mu_0 = 4\pi \times 10^{-7}\ [H/m], \quad c = 1\big/\sqrt{\varepsilon_0\ \mu_0} = 3 \times 10^{8}\ [m/s]$

Problem 1: Three lossless transmission lines having the same phase constant (β) are connected as shown below. What is the input impedance as seen at AA' terminal? Show all your work **(30 Points)**



* The segment CD is λ/2 long so the input impedance seen at CC' is the same as the load Z_L ⇒ $\boxed{Z_{in}^{CC'} = Z_L = 60 - j35\ \Omega}$

* The segment BC is 3λ/4 so from our note we know that for a 3λ/4 line $Z_{in} = R_0^2/Z_L$. In our case then $\boxed{Z_{in}^{BB'} = \frac{(R_{02})^2}{Z_L} = \frac{(100)^2}{60-j35}}$ ⇒

$$\boxed{Z_{in}^{BB'} = 124.352 + j72.539 = 143.963\, e^{j30.26^\circ}}$$

* The input impedance at AA' then is calculated from $Z_{in}^{AA'} = R_{01}\dfrac{Z_{in}^{BB'} + jR_{01}\tan(\beta l)}{R_{01} + jZ_{in}^{BB'}\tan(\beta l)}$

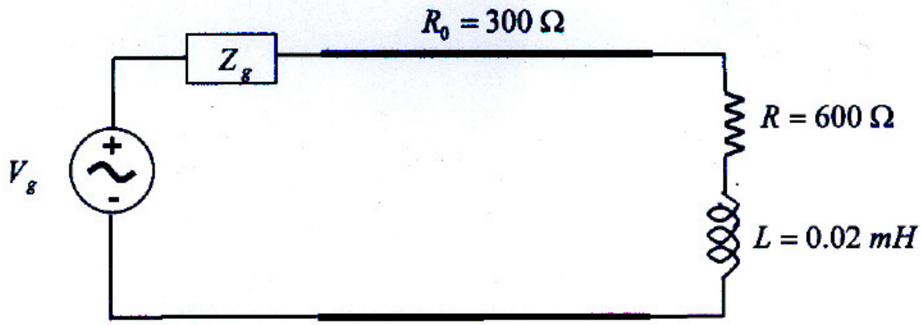
For AB segment $\beta l = \frac{2\pi}{\lambda}\frac{5\lambda}{8} = \frac{5\pi}{4} = \pi + \pi/4$

Hence $\tan(\beta l) = \tan(\pi + \pi/4) = \tan \pi/4 = 1$ then

$$\boxed{Z_{in}^{AA'} = 50\,\frac{(124.352 + j72.539) + j50 \times 1}{50 + j(124.352 + j72.539)\times 1} = 38.929 - j57.056 = 69.072\, e^{-j55.7^\circ}}$$

Problem 2: A 300 Ω lossless air (vacuum) transmission line is connected to a complex load composed of a resistor in series with an inductor, as shown in the figure. At 5 *MHz*, Determine:

a) Reflection coefficient at the load. **(5 Pts)**
b) Standing wave ratio. **(4 Pts)**
c) Location of voltage maximum nearest to the load. **(8 Pts)**
d) Location of current maximum nearest to the load. **(8 Pts)**
e) The values of the distributed capacitance (C) and inductance (L) associated with the transmission line. **(10 Pts)**



a) $\Gamma_L = \dfrac{Z_L - R_0}{Z_L + R_0}$; $Z_L = R + j\omega L$, $\omega = 2\pi \times 5\times10^{6}\ Hz = \pi\times10^{7}\ Hz$

$$Z_L = 600 + j\pi\times10^{7}\times0.02\times10^{-3} = 600 + j200\pi = 600 + j628.319$$

then

$$\Gamma_L = \frac{600 + j200\pi - 300}{600 + j200\pi + 300} = 0.552 + j0.313 = 0.634e^{j29.56^{\circ}}$$

b) $S = \dfrac{1+|\Gamma|}{1-|\Gamma|} = \dfrac{1+0.634}{1-0.634} = 4.464$

c) For $|V|_{max}$ we have $\theta_\Gamma - 2\beta z_{max} = -2n\pi$ $n = 0, 1, 2, 3$

$$\Rightarrow \theta_\Gamma - 2\frac{2\pi}{\lambda} z_{max} = -2n\pi \Rightarrow$$

$$\theta_r + 2n\pi = \frac{4\pi}{\lambda} z_{max} \Rightarrow$$

① $$\frac{\lambda}{4\pi}(\theta_r + 2n\pi) = z_{max}$$ for our case

② $$\boxed{\theta_r = \frac{29.56^\circ \times \pi}{180} = 0.164\pi} \Rightarrow$$

$\lambda = \frac{2\pi}{\beta}$ & $V_p = \frac{\omega}{\beta} \Rightarrow \beta = \frac{\omega}{V_p} = \frac{2\pi v}{c}$ when $V_p = c = 3\times10^8$ m/s

then $$\boxed{\lambda = \frac{\cancel{2\pi} c}{\cancel{2\pi} v} = \frac{c}{v} = \frac{3\times10^8}{5\times10^6} = 60 \text{ m}}$$

from (1) & (2) $$\frac{60}{4\pi}(0.164\pi + 2n\pi) = z_{max} \quad n = 0, 1, 2$$

nearest maximum to load is at $n = 0$ ⇒

$$\boxed{z_{max} = \frac{60}{4\cancel{\pi}} \times 0.164\cancel{\pi} = 2.46 \text{ meter}}$$

d) The location of current maximum is given by

$$\theta_r - 2\beta z_{max}^{(I)} = -(2n+1)\pi \Rightarrow$$

$$\frac{\lambda}{4\pi}(\theta_r + (2n+1)\pi) = z_{max}^{(I)}$$ ⇒ for nearest current max $n = 0$

$$\Rightarrow \frac{60}{4\cancel{\pi}}(0.164\cancel{\pi} + \cancel{\pi}) = z_{max}^{(I)} \Rightarrow \boxed{z_{max}^{(I)} = 17.46 \text{ meter}}$$

* Note you also could have guessed the $z_{max}^{(I)}$ by stating that voltage max & current max are $\lambda/4$ apart ⇒

If voltage max is at $z_{max}^{(V)} = 2.46$ then current max is at $z_{max}^{(I)} = 2.46 + \frac{\lambda}{4} = 2.46 + \frac{60}{4} = \boxed{17.46 \text{ meter}}$

e) we note $V_p = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{LC}} = \frac{1}{\sqrt{LC'}}$, here $V_p = c = 3\times10^8 \Rightarrow$

$c = \frac{1}{\sqrt{LC'}} \Rightarrow \boxed{LC' = \frac{1}{c^2}}$ (1) also $Z_0 = \sqrt{\frac{L}{C}} \Rightarrow \boxed{Z_0^2 = \frac{L}{C'}}$ (2)

(3)

(1) $\Rightarrow L = \frac{1}{c^2 C'}$ use (3) in (2) $\Rightarrow Z_0^2 = \frac{1}{c^2 C' C'} = \frac{1}{c^2 C'^2} \Rightarrow$

$$\boxed{C' = \frac{1}{c Z_0} = \frac{1}{3\times10^8 \times 300} = 1.11\times10^{-11} \text{ F/m}}$$

& from (3)

$$\boxed{L = \frac{1}{c^2 C'} = \frac{1}{(3\times10^8)^2 \times 1.11\times10^{-11}} = 1\times10^{-6} \text{ H/m}}$$

Problem 3: The wave equation for instantaneous voltage in a general transmission line is given by

$$\frac{\partial^2 v(z,t)}{\partial z^2} = LC\frac{\partial^2 v(z,t)}{\partial t^2} + (RC+LG)\frac{\partial v(z,t)}{\partial t} + RG\, v(z,t).$$

a) What is the wave equation for the instantaneous voltage in the case of lossless line? **(5 Pts)**

b) For the equation in part (a) we propose the following solution, $v(z,t) = f(t)\,g(z)$, where $f(t)$ and $g(z)$ are only functions of time and distance, respectively. What are the differential equations governing the behavior of $f(t)$ and $g(z)$? Show all your work and clearly state all your assumptions. **(20 pts)**

c) What are the possible solutions to the differential equation for $g(z)$, found in part (b)? **(5 pts)**

d) How do your results here compare to or relate to what we have studied in the class? **(5 pts)**

a) For lossless line $R = G = 0 \Rightarrow$

$$① \quad \frac{\partial^2 v(z,t)}{\partial z^2} = LC\frac{\partial^2 v(z,t)}{\partial t^2}$$

b) Let $v(z,t) = f(t)\,g(z)$ ②, sub (2) in (1) $\Rightarrow$

$$f(t)\frac{d^2 g(z)}{dz^2} = LC\, g(z)\frac{d^2}{dt} f(t) \Rightarrow \boxed{\frac{1}{g(z)}\frac{d^2 g(z)}{dz^2} = \frac{LC}{f(t)}\frac{d^2 f(t)}{dt^2}} \quad ③$$

For (3) to be true of all t's & z's each term on (3) must be equal to a constant (say m) where by constant we mean m is independent of t & z, then

$$\frac{1}{g(z)}\frac{d^2 g(z)}{dz^2} = m \Rightarrow \boxed{\frac{d^2 g(z)}{dz^2} = m\, g(z)} \quad ④ \quad \&$$

$$\frac{LC}{f(t)}\frac{d^2 f(t)}{dt^2} = m \Rightarrow \boxed{\frac{d^2 f(t)}{dt^2} = \frac{m}{LC} f(t)} \quad ⑤$$

(6)

c) For $\frac{d^2 g(z)}{dz^2} = m\,g(z)$ ju $g(z)$ can be given by $g(z) = A_0 e^{\sqrt{m}\,z}$

or $g(z) = B_0 e^{-\sqrt{m}\,z}$ or in general

(7) $$\boxed{g(z) = A_0 e^{\sqrt{m}\,z} + B_0 e^{-\sqrt{m}\,z}}$$ where A_0 & B_0 are constant

with respect to z.

d) In class we have seen that wave equation for lossless line

for phasor $\bar{V}(z)$ was given by

(8) $$\frac{d^2 \bar{V}(z)}{dz^2} = -\beta^2 \bar{V}(z)$$ from (8) & (6) we see that our

m here is $m = -\beta^2 \Rightarrow$ (8) $\boxed{\sqrt{m} = j\beta}$ & hence Eq (7)

Can be written as

(9) $$\boxed{g(z) = A_0 e^{j\beta z} + B_0 e^{-j\beta z}}$$ which is the same

we have found in class with $A_0 = V_0^+$ & $B_0 = V_0^-$

Remark: Show that solutions to (5), i.e. $f(t)$ can also be written as

$f(t) = C_0 e^{j\omega t} + D_0 e^{-j\omega t}$ where C_0 & D_0 are constant