Negative Group Velocity in Left-Handed Materials

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1. Introduction

In 1968 Veselago posed that media with simultaneous negative values of the permittivity and permeability would exhibit a negative index of refraction [1]. Due to the left-handed relationship between the electric field, magnetic field, and the wave vector in these materials he termed them left-handed media (LHM). These LHM are expected to have many unusual properties such as inverted Snell's law, Doppler shift and Cherenkov radiation. However, until recently, such materials were unavailable and these predictions remained uncorroborated for nearly thirty years.

As with any new field, the topic of the LHM suffers from some misconceptions and misunderstandings. One such misconception has to do with the notion of negative group velocity which has been closely linked with these media [2] and in fact has been used as equivalent to the term LHM [1]. In this presentation the existence and the meaning of negative group velocity in LHM are discussed. It is shown that similar to normal right-handed media (RHM), LHM can also exhibit a negative group velocity and group delay within the region of anomalous dispersion ($\partial n / \partial \omega < 0$). On the other hand, in the experiments presented so far, the researchers have only measured positive group velocity and negative phase velocity corresponding to so called backward waves (BWs).

2. Negative Group Velocity for a Slab with Negative Index of Refraction

Consider a medium having effective electric and magnetic responses characterized by

$$\varepsilon_{eff} = 1 - \frac{\omega_{ep}^2 - \omega_{eo}^2}{\omega^2 - \omega_{eo}^2 + j\gamma_e \omega}, \tag{1}$$

and

$$\mu_{eff} = 1 - \frac{\omega_{mp}^2 - \omega_{mo}^2}{\omega^2 - \omega_{mo}^2 + j\gamma_m \omega}, \tag{2}$$

where ω_{ep}, ω_{mp} are the electric and magnetic plasma frequencies and ω_{eo}, ω_{mo} are the electric and magnetic resonance frequencies respectively. The γ_e and γ_m are the phenomenological electric and magnetic damping constants. Figure (1) shows the real and imaginary parts of the effective index calculated from

$n_{eff} = \sqrt{\varepsilon_{eff}}\,\sqrt{\mu_{eff}}$. Figure 1 shows that for the parameters considered the index of refraction and hence the phase velocity is negative for $19.1 < f < 24.9(GHz)$.

For a slab of thickness *L,* having the above electric and magnetic responses the transmission function (magnitude and phase) can be calculated according to

$$T(\omega) = \frac{t_{12} t_{21} e^{ik_2 L}}{1 - r_{12}^2 e^{i2k_2 L}} = |T(\omega)| e^{j\phi(\omega)}, \tag{3}$$

where $t_{i,j}$ and $r_{i,j}$ are the Fresnel transmission and reflection coefficients corresponding to the slab boundaries and $k_2 = 2\pi/\lambda_o\, n_2 \cos\theta_2$. In the following we have assumed that the LHM is surrounded by vacuum and is illuminated at normal incidence ($\theta_1 = \theta_2 = 0$).

The velocity by which the peak of a well-behaved wave packet travels through the slab, i.e. the packet group velocity, is then given by

$$V_g = \frac{L}{-\partial\phi/\partial\omega} = \frac{L}{\tau_g}, \tag{4}$$

where $\phi(\omega)$ is the phase of the transmission function, and $\tau_g = -\partial\phi/\partial\omega$ is the group delay. For the matched slab, where the interface effects are negligible, Eq. (4) can further be simplified to the well-known expression

$$V_g = \frac{c}{n + \omega\ dn/d\omega} = \frac{c}{n_g}. \tag{5}$$

From Eq. (4) it is clear that the group velocity and group delay have the same sign, hence in studying the negative or positive group velocity we can equally consider the sign of the group delay.

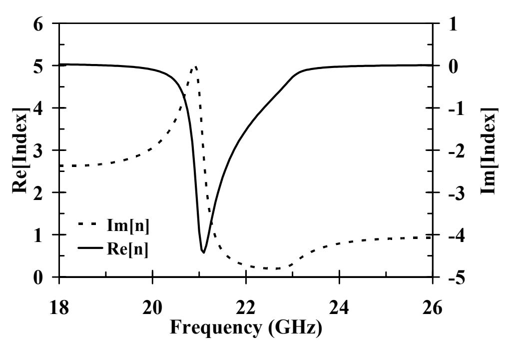

Figure 1. Real and imaginary parts of the index of refraction. The parameters used in obtaining figure 1 and figure 2 are $\omega_{eo} = 0$, $\omega_{mo} = 2\pi \times 21$, $\omega_{ep} = 2\pi \times 40$, $\omega_{mp} = 2\pi \times 23$ (GHz), $\gamma_e = 2 \times 10^9$, and $\gamma_m = 10^9$ (1/s) respectively.

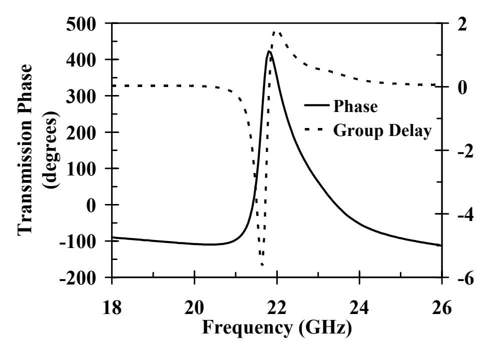

Figure 2. Calculated transmission phase and group delay for a 5mm thick LHM slab immersed in vacuum.

Figure 2 shows the transmission phase and the group delay for a slab 5mm long, with an effective index depicted in Fig. 1. As Fig. 2 clearly indicates, the group delay and hence the group velocity is negative for $20.4GHz < f < 21.8GHz$

(region of anomalous dispersion) and positive outside this region (region of normal dispersion). This change in the sign of the group delay is also evident from the change in the slope of the transmission phase. The fact that the group velocity and hence the group delay are negative in the region of anomalous dispersion can also be verified from Fig. 1 and Eq. (5). Within this region $\omega\, dn/d\omega$ and n are both negative implying a negative value for the group index (n_g) and hence the group velocity. On the other hand, away from the region of anomalous dispersion $\omega\, dn/d\omega$ is positive and larger than n, implying a positive value for the group index (n_g) and the group velocity.

In light of references to the existence of negative group velocity in the LHM it must be noted that the negative group velocity has a well-defined physical meaning. For a medium with negative group velocity, the peak of the transmitted wave packet would emerge prior to the peak of the well-behaved incident wave packet entering the medium [3] [4]. It must be added that such counter intuitive behavior is not in conflict with the requirements of special relativity and causality since in all cases the "true" information conveyed by the wave packet "front" suffers a positive and causal delay [3].

3. Full Wave Simulation for LHM with Split Ring Resonator and Strip Wires

Thus far, for the sake of simplicity, we have only considered the case of a slab having a dispersive negative index of refraction. Let us now consider a commonly studied LHM consisting of structures known as the Split Ring Resonators (SRRs) and Strip Wires (SWs) or metallic rods. In this presentation we will discuss the transmission magnitude, transmission phase, and the group delay for such media. Our full wave simulations (finite element method) of the experimental results presented in [2] show that, in general, these structure also support a negative group velocity in their anomalous dispersion regions. However, to date, the researchers have focused on measuring the transmission magnitude and in particular the transmission within the region of normal dispersion. In such, their results are exclusive to the positive group velocity (pointing away from the source) and negative phase velocity (pointing toward the source). Therefore, describing these structures as negative group velocity media is inappropriate. More suitably, the dynamics of the wave propagation in these situations can be described in terms of BWs.

Figure 3 shows the transmission phase and the group delay for a LHM consisting of 7 square SRRs and SWs. In order to make possible the comparison with an experiment currently under way, the structure was designed to operate in the K-band. The structure displays both positive and negative group delays and hence positive and negative group velocities. It must be added that the negative group velocity within the anomalous dispersion region is inherently narrow-band and is accompanied by strong absorption. The band-width and the negative value of the delay can be augmented by increasing the LHM thickness as shown in Figure 4. However, this means more absorption for the propagating wave packets. Greater negative values for the group delay mean that the peak of the transmitted wave packet is more advanced in time as compared to the incident wave-packet, however, at the price of greater reduction in its magnitude.

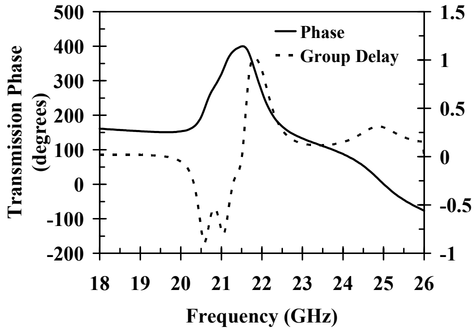

Figure 3. Transmission phase and group delay for a 7 element array of SRRs and SWs. The ring dimensions were scaled down from those in [2] for operation in the K-band. The strips are 0.2mm wide with $\sigma = 5.8 \times 10^6$ *S/m*.

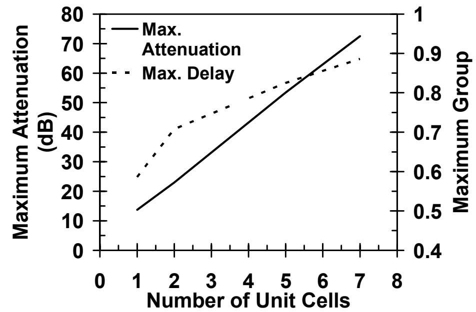

Figure 4. Negative group delay and attenuation as function of the number of SRRs and SWs.

4. Conclusions

We have shown using both full wave simulations and analytical techniques that in LHM the phase velocity remains negative over the negative index range but the group velocity can be positive (backwards wave propagation) or negative. This negative group velocity (or group delay) is a result of propagation in the region of anomalous dispersion as has been demonstrated in [5]. Increasing the length of the LHM can increase the negative value of the delay and enhance the bandwidth of the negative group delay region. However, a thicker LHM will result in more attenuation, therefore a practical limit must be set as to how much bandwidth is required and how much attenuation can be tolerated.

References

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