Left-Handed and Right-Handed Metamaterials composed of Split Ring Resonators and Strip Wires

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I. Introduction

In 1967 Veselago stated that a medium with negative real parts of the electric permittivity and magnetic permeability would have a negative real index of refraction [1]. In such a medium the electric field, the magnetic field, and the propagation vector form a left-handed (LH) triplet. Because of this relationship, these media have been named left-handed media (LHM). The first negative index medium was developed three decades later when Smith combined an array of split ring resonators (SRRs) and strip wires (SWs) [2]. Typically, in such a medium, in order to determine whether the given structure has a negative index of refraction or not, the transmission through the structure is measured and if a transmission peak is observed in the region where both the real parts of the permittivity and permeability are expected to be negative, then the structure is assumed to exhibit LHM behavior [2, 3, 4]. However the emergence of a passband and a transmission peak is not sufficient evidence that the structure exhibits left handedness. When the SRR and SW are combined into a single structure their individual field patterns can interfere in such a way that the LHM behavior is weakened or even lost, whereas still a passband and a transmission peak can be observed. In these situations, in order to correctly predict the left or right handedness, one must rely on other “diagnostic tools” such as calculating or measuring the *insertion phase and/or dispersion diagrams*.

II. Transmission Magnitude

Consider two structures composed of square SRRs and SWs. In the first case the rings and strips are printed on the same side of the substrate and in the second case the rings and strips are printed on opposite sides of the substrate. We refer to these as the same side (SS) and opposite side (OS) structures respectively. The dimensions of the rings and strips in both structures were the same and are given in [5]. The transmission through configurations such as those mentioned above has been simulated and measured experimentally by many researchers [3, 5, 6]. In most cases where such structures have been considered a transmission peak has been observed, and hence it has been concluded that a negative index of refraction exists.

We have simulated the transmission through the two SRR and SW configurations using Ansoft HFSS as shown in Fig. 1. For both structures there is a peak in the transmission magnitude where both the real part of the electric permittivity and the real part of the magnetic permeability are expected to be negative. The peaks appear at 22.6 GHz and 22.75 GHz in the OS and SS cases respectively. Before coming to the conclusion that both these structures are LHM, let us investigate the problem more closely and consider the transmission phases and dispersion relations for the two cases.

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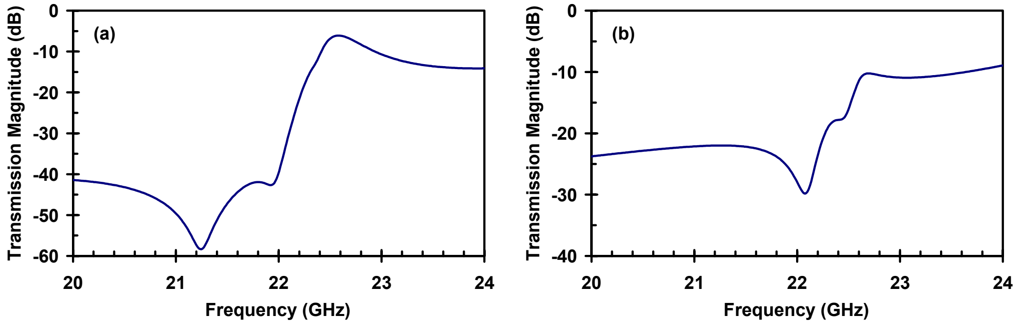

FIG. 1. Simulated transmission magnitude through 4 unit cell long (a) OS structure, and (b) SS structure.

III. Transmission Phase

The sign of the effective index in a structure can be determined by considering the difference in the transmission phase for propagation through various lengths of the same structure. For a medium with phase index $n(\omega)$ the difference in transmission phase for propagation through lengths L_1 and L_2 is given by

$$\Delta\phi = -n(\omega)(L_2 - L_1)\frac{\omega}{c} \tag{1}$$

where c is the speed of light in vacuum. For $L_2 > L_1$ this difference will be negative in RHM [$n(\omega) > 0$], and positive in LHM [$n(\omega) < 0$] case. The transmission phases for propagation through 1, 2, 3, and 4 unit cells of the OS and SS configurations are shown in Fig. 2.

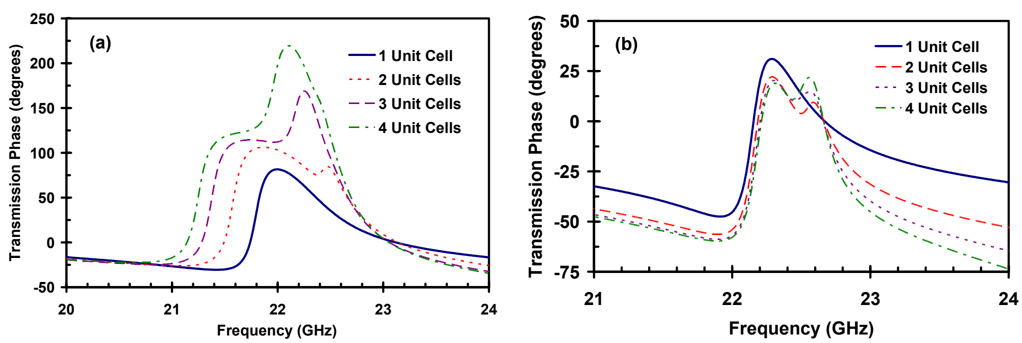

FIG. 2. Transmission phase for propagation through 1, 2, 3, and 4 unit cells of (a) OS structure and (b) SS structure.

In Fig. 2(a) the phase difference ($\Delta\phi$) is positive between 20.7 GHz and 22.9 GHz, implying that the effective index is negative in this region. At 22.9 GHz the phase lines cross, signifying the transition from LHM to RHM behavior. In Fig. 2(b) the phase difference is negative from 21 GHz to 22.4 GHz, so that the effective index is positive in this region. At 22.4 GHz the phase lines for the two, three, and four unit cell cases cross indicating a transition for these cases to LHM behavior. The one unit cell case crosses and makes its transition to LHM behavior at 22.57 GHz. At 22.65 GHz all four cases return to RHM behavior. This difference in the behavior of the single and multiple unit cell cases is caused by the interactions between nearest neighbor SRRs. What is important to note is that in the OS case the LHM region (20.7 – 22.9 GHz) contains the transmission peak (22.6 GHZ) shown in Fig. 1(a), whereas in the SS case the transmission peak (22.75 GHz) lies outside the LHM region (22.4 – 22.65 GHz). *Hence, in the passband, the OS structure is a LHM and the SS structure is a RHM.*

IV. Dispersion Diagrams

The dispersion characteristics of the SS and OS structures were also simulated using both Ansoft-HFSS and a finite difference time domain algorithm (developed in-house) and are shown in Fig. 3 (solid lines). When the mode resulting from the SRRs and SWs intersects the light line (dashed line) there is a coupling. Hence, for comparison, the dispersion plot was also calculated using an equivalent transmission line model for the OS case which does not take this coupling into account (dotted line).

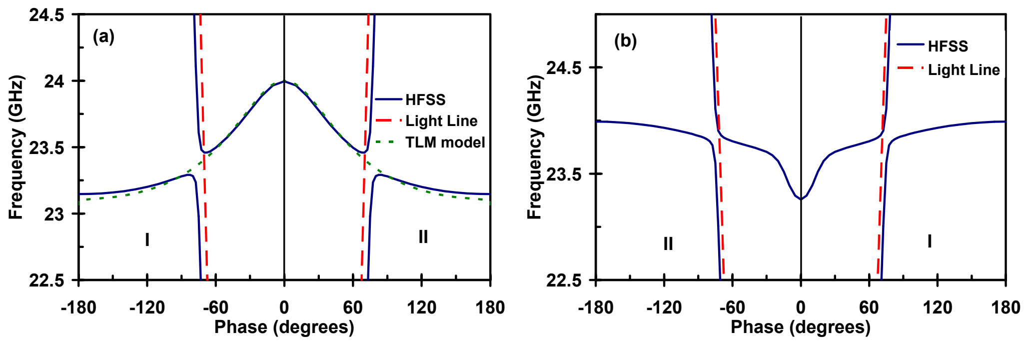

FIG. 3. Dispersion plots for (a) OS structure and (b) SS structure. In the OS configuration the unit cell size was 2.5×2.5×2.5mm. The unit cell size was 4×2.5×2.5mm in the SS case.

For the case of one dimensional propagation considered here, the local derivative of the curves depicted above is the group velocity or equally the energy velocity (in the passband), so that only branches marked (I) in Figs. 3(a) and 3(b) correctly predict positive energy propagation [5]. Note that in Fig. 3(a) while the local derivative (the group velocity) is positive within branch-I, the slope of the line joining the origin to any point on branch-I is negative, indicating a negative phase velocity. In other words, this mode exhibits backward wave behavior indicative of an effective negative index of refraction. On the other hand, the propagating mode of the SS structure depicted by branch-I of Fig. 3(b) has both positive group and phase velocities, indicating the existence of a positive index of refraction. These dispersion diagrams corroborate the results from Section III that, in the passband, the OS structure is a LHM and the SS structure is a RHM. The LHM and RHM character of these structures can also be seen by the manner in which they couple to the light line. In the OS case the coupling results in the emergence of a stop band, which is characteristic of contra-directional coupling (RHM-LHM). In the SS structure, on the other hand, no stop band appears, characteristic of co-directional coupling (RHM-RHM).

V. The Effects of Unit Cell Size

By reducing the size of the unit cell the strength of both the SRR and SW resonances can be enhanced. Thus, even though in the previous case the SS structure did not exhibit a LHM behavior in the pass-band, by reducing the unit cell size it can be made to display left handed behavior. Fig. 4 shows several band diagrams for the SS structure obtained by varying the unit cell size in the direction parallel to the axis of the SRRs (i.e., perpendicular to the direction of propagation.)

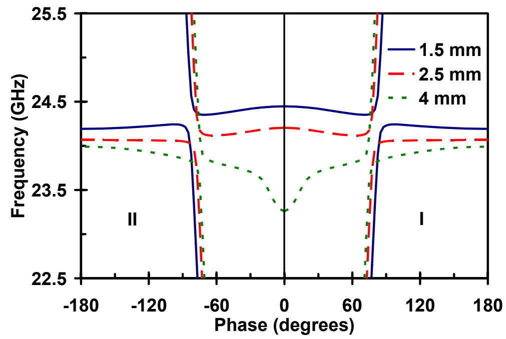

FIG. 4. Dispersion curves for the SS structure.

In Fig. 4, when the unit cell is 4mm long in the direction parallel to the ring axis (dotted) the slope of the dispersion curve is positive everywhere for the branch marked-I, indicative of positive group and energy velocities. When this dimension is decreased from 4 mm to 2.5 mm (dashed line) the passband is nearly flat. However, further reduction of the unit cell size to 1.5 mm results in an LHM band with the correct behavior described by branch-II (solid line). Thus, reducing the unit cell size allows the structure to make a transition from RHM to LHM behavior. An examination of Fig. 4 also reveals that the coupling makes a transition from co-directional coupling at 4 mm to contra-directional coupling at 1.5 mm.

VI. Summary and Concluding Remarks

It was shown that the existence of transmission peaks in the regions where the real parts of the electric permittivity and magnetic permeability are expected to be simultaneously negative is not sufficient evidence of LHM behavior. In order to properly determine the sign of the index, the insertion phase for propagation through several lengths of the structure is necessary. Alternately, the dispersion diagrams can be used to determine the sign of the index. In addition, it was shown that both the OS and SS configurations can produce LHM behavior, but that in the SS case, because of the interference between the SRRs and SWs, the lattice spacing must be smaller than that of the OS case. In other words, the interference between the SRRs and SWs in the SS case does not completely destroy the potential for LHM behavior, it weakens it.

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