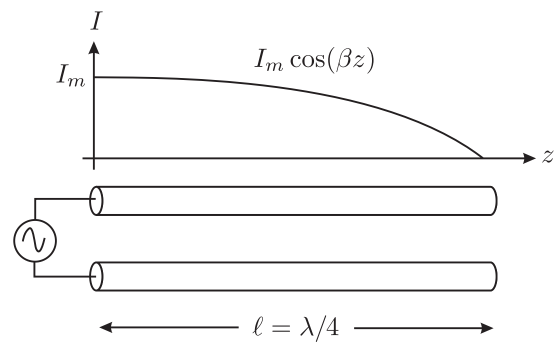
# Half-wave Dipole

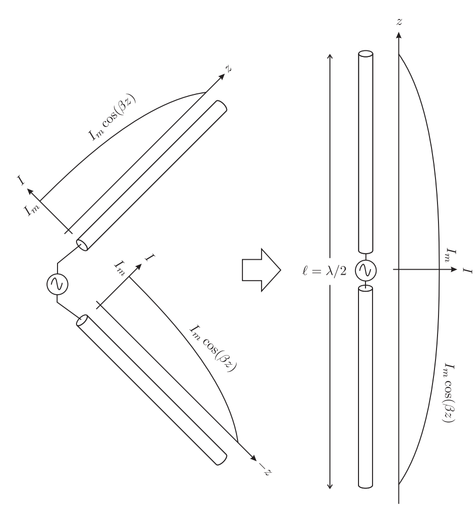
The dipole we have studied so far is not terribly useful, since it is not very efficiency and difficult to impedance-match to. Both these facts are a result of the electrically small nature of the antenna. A more practical dipole is the half-wave dipole (referring to the fact that it is $\lambda/2$ long). The main reason for this, as we will see, is that the half-wave dipole has a real input impedance at resonance which is close to common system impedances.

## Radiated Fields

Given the length of the dipole, it seems doubtful that the current distribution will be uniform as with the case of the Hertzian dipole. If we think about an open-circuited transmission line made of two wires, we imagine a sinusoidal current distribution set up by the standing wave along a quarter-wavelength length of line as follows:




Note that there is no current at $z = \lambda/4$ as required by the open circuit boundary condition. Now, if we "open" up the transmission line, we can essentially create a dipole that is half a wavelength long:




We can write the current distribution as

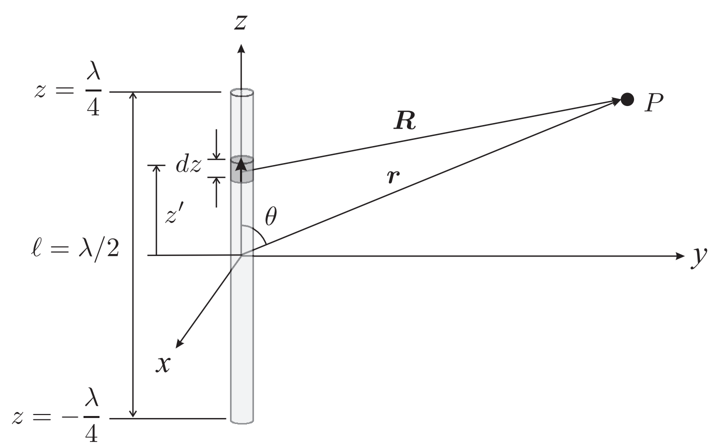
$$I(z) = I_m \cos(\beta z), \tag{1}$$

where $\beta$ is the phase constant associated with the transmission line from which we have drawn the current distribution. Since we are in free space, $\beta = \omega/c = k$. Knowing the current distribution, our next question is how to find the electric field produced by the dipole? Well, we know that a tiny piece of dipole produces an electric field in the far field of

$$E_\theta = \frac{I\Delta z j\omega\mu}{4\pi}\frac{e^{-jkr}}{r}\sin\theta \tag{2}$$

if excited with a current element of amplitude I at the origin. Using superposition, we can represent the half-wave dipole as a collection of Hertzian dipoles and add up all the responses of each dipole. Hence, each dipole "piece" contributes an electric field

$$dE_\theta = \frac{I(z')dz' j\omega\mu}{4\pi}\frac{e^{-jkR}}{R}\sin\theta. \tag{3}$$




Now, since we are in the far field, the diagram above is not really correct. As the point P moves far from the source, the vectors $\boldsymbol{R}$ and $\boldsymbol{r}$ become parallel. This is known as the *parallel ray approximation*. Under this approximation,

$$\frac{1}{R} \approx \frac{1}{r} \text{ for amplitude variations} \tag{4}$$

$$\exp(-jkR) \approx \exp[-jk(r - z'\cos\theta)] \text{ for phase variations} \tag{5}$$

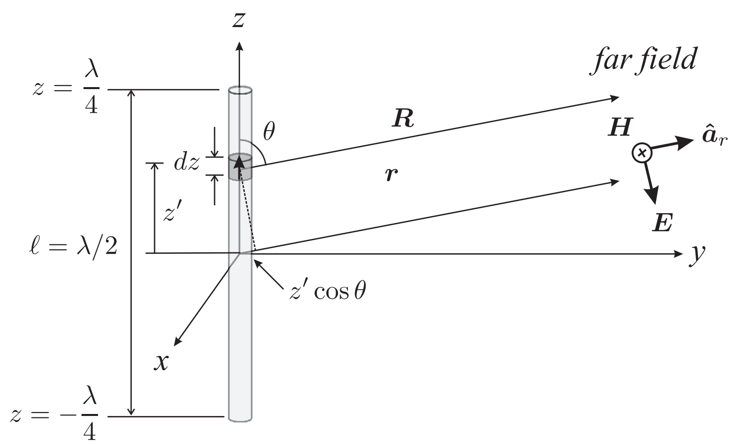
where the latter approximation is evident by examining the geometry of the far-field situation.

Then,

$$dE_\theta = \frac{I(z')dz' j\omega\mu}{4\pi}\frac{e^{-jkr}}{r}e^{jkz'\cos\theta}\sin\theta \tag{6}$$

$$E_\theta = \int_{z'=-\lambda/4}^{z'=\lambda/4}\frac{I(z')j\omega\mu}{4\pi}\frac{e^{-jkr}}{r}e^{jkz'\cos\theta}\sin\theta dz' \tag{7}$$

$$= \frac{j\omega\mu}{4\pi}\frac{e^{-jkr}}{r}\int_{z'=-\lambda/4}^{z'=\lambda/4} I_m\cos(kz')e^{jkz'\cos\theta}\sin\theta dz' \tag{8}$$




---

NOTE:

$$\int \sin(a+bx)e^{cx} = \frac{e^{cx}}{b^2+c^2}[c\sin(a+bx) - b\cos(a+bx)] + C \tag{9}$$

$$\begin{aligned}
\int_{z'=-\lambda/4}^{z'=\lambda/4} \sin(\pi/2+kz')e^{jkz'\cos\theta} &= \frac{e^{jkz'\cos\theta}}{k^2+(jk\cos\theta)^2}\left[jk\cos\theta\sin(\pi/2+kz') - k\cos(\pi/2+kz')\right]\Big|_{-\lambda_4}^{\lambda/4} \\
&= \frac{e^{jk\frac{\lambda}{4}\cos\theta}}{k^2-k^2\cos^2\theta}\left[jk\cos\theta\cancel{\sin(\pi/2+k\lambda/4)} - k\cos(\pi/2+k\lambda/4)\right] - \\
&\quad \frac{e^{-jk\frac{\lambda}{4}\cos\theta}}{k^2-k^2\cos^2\theta}\left[jk\cos\theta\cancel{\sin(\pi/2-k\lambda/4)} - k\cos(\pi/2-k\lambda/4)\right] \\
&= \frac{e^{j\frac{\pi}{2}\cos\theta}}{k^2\sin^2\theta}k + \frac{e^{-j\frac{\pi}{2}\cos\theta}}{k^2\sin^2\theta}k = 2\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{k\sin^2\theta}
\end{aligned} \tag{10}$$

---

Therefore,

$$E_\theta = \underbrace{\frac{j\omega\mu I_m}{4\pi}\frac{e^{-jkr}}{r}\sin\theta}_{\text{Hertzian dipole E-field}} \cdot \underbrace{2\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{k\sin^2\theta}}_{\text{space factor}} \tag{11}$$

and since $\omega\mu/k = \eta$,

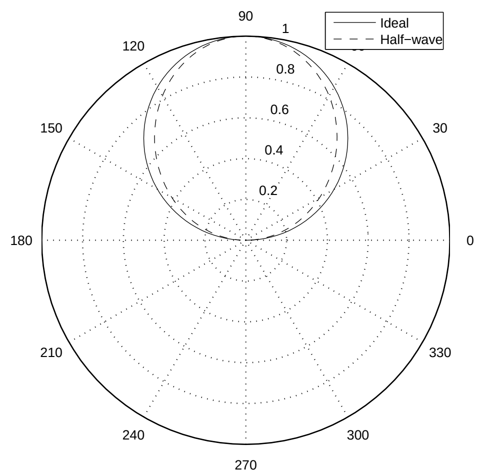
$$E_\theta = \frac{j\eta I_m}{2\pi}\frac{e^{-jkr}}{r}\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}. \tag{12}$$

$H_\phi$ follows as

$$H_\phi = \frac{E_\theta}{\eta} = \frac{jI_m}{2\pi}\frac{e^{-jkr}}{r}\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}. \tag{13}$$

# Radiation Pattern

If we take a polar plot of the pattern indicated by the above expressions, and compare to the pattern from a Hertzian dipole, we notice that a half-wave dipole has slightly less beamwidth than the Hertzian dipole. In fact, the HPBW of a Hertzian dipole is $90°$, while that of a half-wave dipole is only $78°$. Hence, we expect the half-wave dipole to exhibit slightly more directivity than its Hertzian counterpart.




# Directivity and Input Impedance

Let's evaluate the directivity and input impedance of the half-wave dipole at the frequency where the dipole is exactly half a wavelength long. We begin by calculating the radiation intensity produced by the dipole:

$$U(\theta) = \frac{1}{2} r^2 \frac{|E_\theta|^2}{\eta} = \frac{1}{2} \frac{\eta I_m^2}{(2\pi)^2} \frac{\cos^2(\pi/2 \cos\theta)}{\sin^2\theta}. \tag{14}$$

The radiated power produced by the dipole is

$$\begin{aligned} W_{rad} &= \int_0^{2\pi} \int_0^{\pi} U(\theta) \sin\theta d\theta d\phi & (15) \\ &= \frac{1}{2}(2\pi) \frac{\eta I_m^2}{(2\pi)^2} \underbrace{\int_0^{\pi} \frac{\cos^2(\frac{\pi}{2}\cos\theta)}{\sin\theta} d\theta}_{1.2188 \text{ numerically}} & (16) \\ &= 30(1.2188) I_m^2 = 36.5640 I_m^2. & (17) \end{aligned}$$

The directivity relative to an isotropic radiator is then calculated as

$$D_m = \frac{4\pi U_m}{W_{rad}} = \frac{4\pi}{8\pi^2} \eta I_m^2 \cdot \frac{1}{36.5640 I_m^2} = 1.64 \tag{18}$$

Therefore,

$$D_{dipole} = 1.64 = 2.15 \text{ dBi} = 0 \text{ dBd}. \tag{19}$$

Notice that the dBd unit expresses the directivity with respect to a half-wave dipole, and hence compared to itself, a half-wave dipole has 0 dBd of gain.

For the input impedance, we anticipate both a real and imaginary part, since the near-fields of the dipole will contribute to a reactive component. The input resistance can be found as follows: Then,

$$R_{rad} = \frac{2W_{rad}}{I_m^2} = 73.1280\ \Omega \tag{20}$$

The calculation of the reactive part of the input impedance is much more involved and beyond the scope of the discussion here. The final result for the dipole's input impedance is

$$Z_{dipole} = 73 + j42.5\ \Omega. \tag{21}$$

That is, the input impedance of the dipole is slightly inductive. However, there exists a "resonance" frequency where the imaginary part of the dipole's input impedance goes to zero. This occurs at a slightly lower frequency and produces

$$Z_{dipole} = 70 + j0\ \Omega, \tag{22}$$

which is a useful operating point for the antenna. Common coaxial lines, such as RG-59U, have a characteristic impedance of $75\ \Omega$ and hence can readily be connected to a dipole without impedance matching, although usually one cannot feed dipoles directly from coaxial line (more on that later).

Finally, the ohmic loss in a half-wave dipole is

$$R_{ohmic} = \frac{R_s}{2\pi a}\frac{\lambda}{4}. \tag{23}$$

The details of this calculation have been omitted, but this is not the same expression as a Hertzian dipole. The reason for this is that the ohmic losses are a function of position because the current is not uniformly distributed along the length of the dipole. In fact, if one plugs in $L = \lambda/4$ into the expression for the Hertzian dipole and compare to the above expression, the ohmic loss is twice that predicted by (23), suggesting that only half of the dipole effectively contributes to significant ohmic losses.