Continuous Arrays

1 One-dimensional Continuous Arrays

Consider the 2-element array we studied earlier where each element is driven by the same signal (a uniform excited array), where the overall array length is D and the elements are separated by d. Consider the case where an additional third element is added to the array mid-way between the two elements, leading to the element distance being halved. Then another two in the spaces midway between those three are added, and so on, as shown below. In the limit as the number of points goes to infinity, the array becomes continuous, and so does the current distribution of the array. Instead of an excitation at discrete points in space, the excitation becomes continuous, as shown in Figure 1.

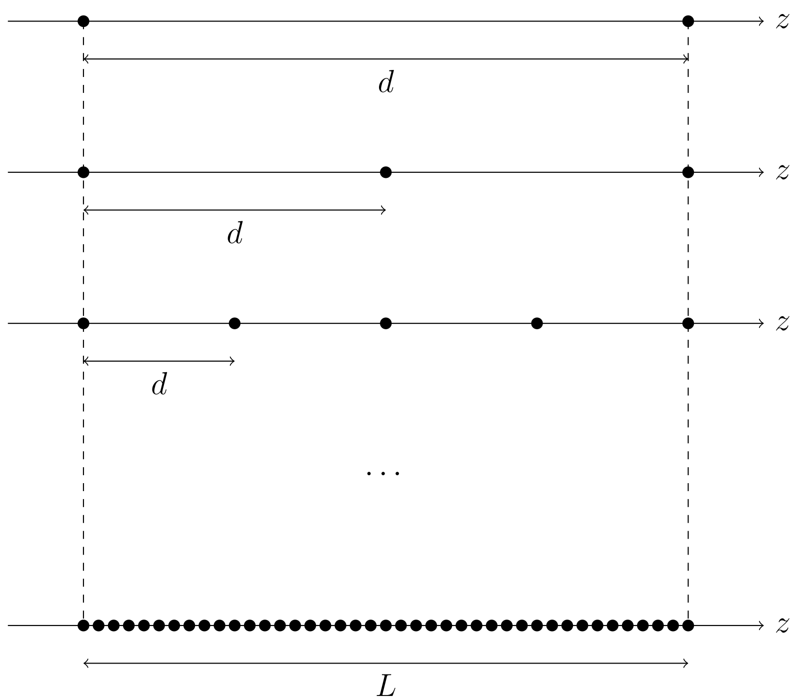

Figure 1: Continuous array

The array factor for a discrete, linear spaced array is

$$AF = \sum_{m=0}^{N-1} I_m e^{jkz\cos\theta} \tag{1}$$

where previously we defined $z = md$. If the array is centred on the z-axis and we let the number of points become infinite, and the array spacing d tends to zero, our summation is replaced with an integral:

$$SF = \int_0^L I(z) e^{jkz\cos\theta} dz = \int_0^L I(z) e^{j2\pi\frac{z}{\lambda}\cos\theta} dz, \tag{2}$$

where $I(z) = 1$ over the interval $0 \leq z \leq L$, and the array factor for the continuous array has been called the *space factor SF*. Since $I(z) = 0$ outside this interval, we can replace the limits of the integral as follows, without loss of generality:

$$SF = \int_{-\infty}^{\infty} I(z) e^{j2\pi \frac{z}{\lambda} \cos\theta} dz. \tag{3}$$

We recognize this as the Fourier Transform of $I(z/\lambda)$ between the spatial domain z/λ and angular domain $u = \cos\theta$. We are used to thinking of Fourier Transforms in terms of time and frequency; think of z/λ as the "spatial" frequency variable and u as the "time variable." $I(z/\lambda)$ defines a pulse that is L/λ in extent, which if centred at the origin is expressed mathematically as

$$I(z/\lambda) = \text{rect}\left(\frac{z/\lambda}{L}\right). \tag{4}$$

We know the Fourier Transform of a pulse is a sinc function, therefore the space factor is

$$SF = \text{sinc}\left(\frac{L}{\lambda}\cos\theta\right). \tag{5}$$

The array factor is plotted in Figure 1 for various array lengths. This kind of current distribution is very useful because as you can see as the length of the distribution increases the directivity of the antenna as a whole becomes extremely high.

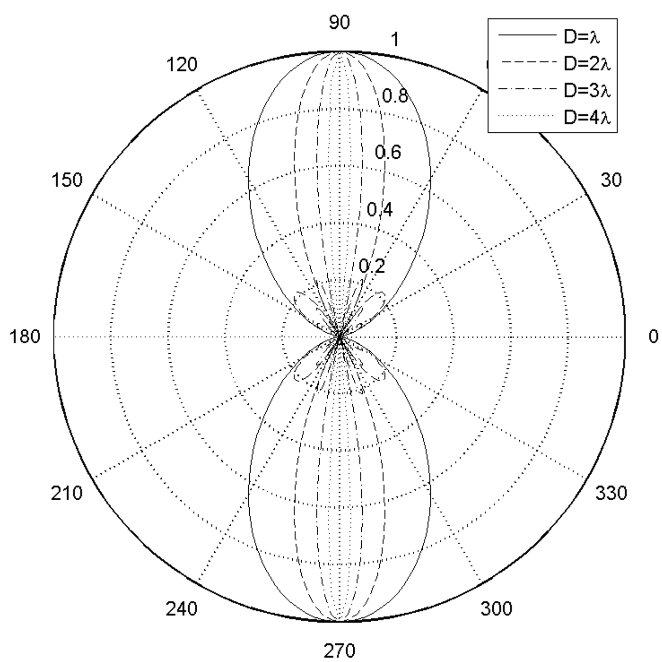

Figure 2: Polar plots of SF for various array lengths; D in the legend denotes the array length L.

The first nulls of the sinc(x) function are at $x = \pm 1$[1]. Therefore, the first nulls in the beam pattern occur at $\frac{L}{\lambda}\cos\theta^{\mp} = \pm 1$. Since the beam maximum as at $\theta = 90^\circ$, and we are very close to $\theta = 90^\circ$ at the first nulls, $\cos\theta^{\mp} = \sin(\pi/2 - \theta^{\mp}) \approx \pi/2 - \theta^{\mp}$. Therefore,

$$\theta^{+} = \frac{\pi}{2} + \frac{\lambda}{L} \quad (6)$$

$$\theta^{-} = \frac{\pi}{2} - \frac{\lambda}{L} \quad (7)$$

and

$$FNBW = \theta^{+} - \theta^{-} = \frac{2\lambda}{L} \quad (8)$$

You can show that the half power beamwidth is approximately half this value (λ/L).

You might think to yourself, "Well, our array has turned into a line source. So, why don't we just use a very long dipole to achieve the same thing as this 'continuous array"'? Well, you may recall that with a dipole we are forced to maintain a sinusoid current distribution along the wire in order to meet the boundary conditions at the end of the wire (namely, that the current must be zero at the ends). Because of the sinusoidal current distribution on a wire, it is impossible to generate the patterns shown for the continuous array; some examples for some dipoles having the same lengths as shown previously are plotted in Figure 1. They have multiple *grating lobes* shooting off in all sorts of directions yielding a very impractical pattern, that on average, has low directivity. Hence, the uniform continuous array has superior properties.

It is not yet obvious how one would synthesize such a current distribution in real life, since we have shown that wires obviously do not support these gate-like current distributions. Such a distribution is support by projecting a smaller, gate-like current distribution onto a larger line. To understand that situation, we need to first generalize to a two dimensional case.

2 Two-dimensional Continuous Arrays

We can construct a two-dimensional continuous array by taking a whole bunch of continuous arrays and stacking them next to each other, an infinitesimal distance apart, to form an *aperture*. Such an aperture is shown in Figure 2.

If the excitation of the aperture is uniform, then we still produce a broadside beam as before, except it has width in both the principal directions of the array elements. In the x-direction, the HPBW is λ/d_1 and in the y-direction, the HPBW in λ/d_2. Such a continuous array is capable of synthesizing a 3-dimensional beam shape of very high directivity, and is called an *aperture antenna*.

We can use the HPBW results to derive the directivity of the array. At a distance r from the aperture, the approximate width of the main beam is $r\lambda/d_1$ in the x-direction, and $r\lambda/d_2$ in the y-direction. Hence the area of the beam is approximately $r^2\lambda^2/d_1 d_2$. Recall the maximum directivity of an antenna is defined as the ratio of the power density radiated in the preferred direction of the antenna to that produced by an isotropic radiator. Therefore, if each antenna

[1] Note sinc(x) $= \sin(\pi x)/(\pi x)$.

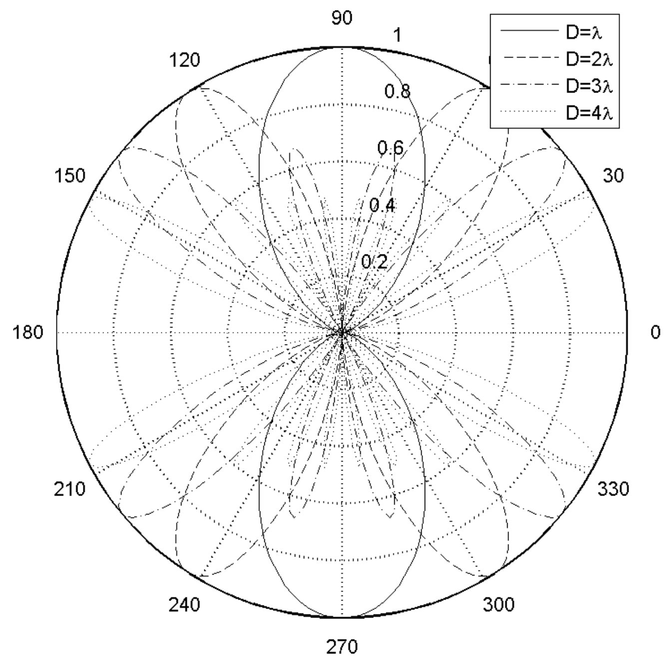

Figure 3: Patterns from dipole antennas of various lengths; D in the legend denotes the dipole length L.

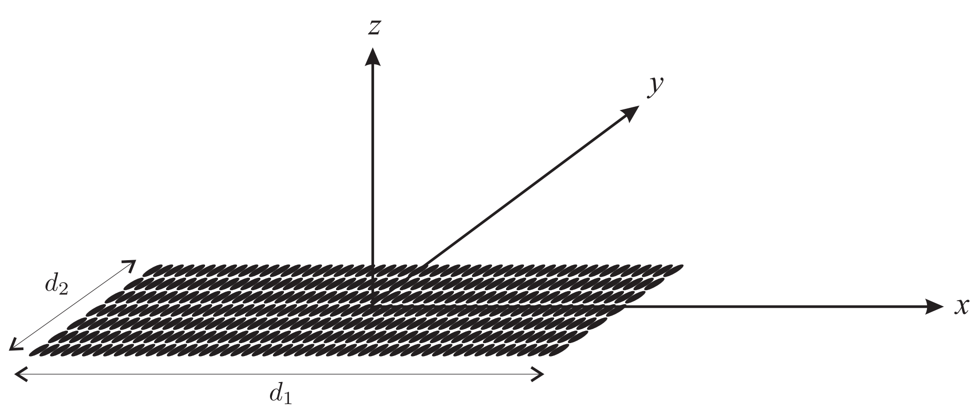

Figure 4: Two-dimensional continuous array

transmits a power W_t, then the power densities produced by the aperture antenna, and isotropic radiator, respectively, are

$$P_{ap} = \frac{W_t}{r^2\lambda^2/d_1d_2} \tag{9}$$

assuming all power is transmitted through the beam area we defined previously (approximate), and

$$P_{iso} = \frac{W_t}{4\pi r^2} \tag{10}$$

(recalling that $4\pi r^2$ is the surface area of a sphere). The directivity is then

$$D = \frac{P_{ap}}{P_{iso}} = \frac{4\pi d_1 d_2}{\lambda^2}. \tag{11}$$

Noting that the area of the aperture is $A_{ap} = d_1 d_2$,

$$D = \frac{4\pi A_{ap}}{\lambda^2}. \tag{12}$$

This is an interesting result since when we compare it to our earlier definition of directivity,

$$D = \frac{4\pi A_{em}}{\lambda^2} \tag{13}$$

and we see the expressions are virtually identical, with the maximum effective area of the antenna replaced with the physical area of the aperture. This is the real advantage of aperture antennas: the gain of the antenna is ultimately only limited by the area of the aperture! In reality, the gain of the antenna may be lower because it is related to directivity by multiplying by e_r, the radiation efficiency of the antenna. Hence, the antenna has an *effective area* of $A_{eff} = e_a A_{ap}$, where e_a is the so-called *aperture efficiency* of the antenna. You can use the fact the the directivity of the antenna is related to its physical area to any aperture shape. For example, for circular apertures of diameter L, $A_{eff} = e_a A_{ap} = e_a \pi (L/2)^2$.

The beamwidths in each principal plane were defined as $\theta_1 = \lambda/d_1$ and $\theta_2 = \lambda/d_2$. Since the beam area is approximately $r\theta_1 r\theta_2 = r^2\theta_1\theta_2$, the directivity expression can be written as

$$D = \frac{\text{sphere area}}{\text{beam area}} = \frac{4\pi r^2}{r^2\theta_1\theta_2}. \tag{14}$$

Hence,

$$\theta_1\theta_2 D = 4\pi \tag{15}$$

can be used as a design equation in general for aperture antennas. In the case of a 2D aperture, we synthesize a uniform excitation in space by using horn antennas (which have a rectangular/circular opening at the end of the antenna). The equivalent current distribution is formed by the fields at the opening in the antenna. The fields can be projected onto larger aperture such as reflectors, allowing us to realize exceptionally high gains from such structures.

The results assume that the aperture field is uniform. In practise, though it is not possible to get a perfectly uniform aperture, we can get close (realistic apertures always have an amplitude taper across them). Perhaps the most important is to realize constant phase across the aperture. Practical apertures are realized by projecting the fields of small apertures (such as the fields in a rectangular waveguide) into a larger physical area. This is accomplished through the use of horn antennas, where the transition obviously serves the purpose of expanding the field to a larger aperture. It can also be done by projecting the field onto a larger reflector, which is shaped to maintain the uniformity of the phase while increasing the physical area of the antenna. The "cantenna" we have looked at in class also achieves a similar effect. The resulting taper is partly responsible for lowering the aperture efficiency e_a of the antenna, defined as being unity when the aperture is uniformly illuminated and no illumination extends past the aperture. Reflector antennas will often have an amplitude taper of 10 dB across the span of the reflector, resulting in an aperture efficiency of around 70%.

3 Specific Aperture Examples

We have seen that

$$\theta_1\theta_2 D = 4\pi \; [sr] \tag{16}$$

if θ_1, θ_2 are in radians, or equivalently

$$\theta_1\theta_2 D = 41,253 \tag{17}$$

if θ_1, θ_2 are in degrees.

Square Aperture

If we use a square aperture, then

$$\theta_1 = \theta_2 \equiv \theta_{sq}. \tag{18}$$

Then,

$$\theta_{sq}^2 D = 4\pi \tag{19}$$

and

$$\theta_{sq}\sqrt{D} = 2\sqrt{\pi} \text{ [rad] or } 203.2 \text{ [deg]}. \tag{20}$$

Circular Aperture

A circular aperture of diameter L has a half-power beamwidth of θ_{cir}. Then a distance r from the aperture, the radius of the subtended beam is

$$a = r\frac{\theta_{cir}}{2}. \tag{21}$$

The area of the beam is therefore πa^2. Then,

$$D = \frac{4\pi r^2}{\pi a^2} = \frac{4r^2}{a^2} = \frac{4r^2}{r^2\frac{\theta^2}{4}} = \frac{16}{\theta_{cir}^2} \tag{22}$$

describes the directivity of the aperture if you know the HPBW. Conversely, we can use the formula

$$D = \frac{4\pi}{\lambda^2}A_{ap} = \frac{4\pi}{\lambda^2}\pi(L/2)^2 = \left(\frac{\pi L}{\lambda}\right)^2 \equiv \frac{16}{\theta_{cir}^2}. \tag{23}$$

Solving yields

$$\theta_{cir} = \frac{4\lambda}{\pi L} \tag{24}$$

which allows us to predict the HPBW knowing the diameter of the aperture.

4 Far Field from Aperture Antennas

Consider a circular aperture, such as an open-ended circular waveguide, which has a diameter D as shown in Figure 4. Close in to the antenna, if the aperture is excited uniformly, it will appear to produce a beam of cylindrical shape; however, we know as the distance from the antenna increases, the fields diverge and a far distance from the antenna the beam shape looks conical, whose half-power angle is given by $\theta_b = \lambda/D$. There is a position r_t where the beam transitions between the near-field cylinder and the far-field cone, as shown in the diagram. Hence, the beam appears to emanate from a point on the aperture known as the *phase centre*, because the phase of a field quantity in the far field can be computed from the distance to this point.

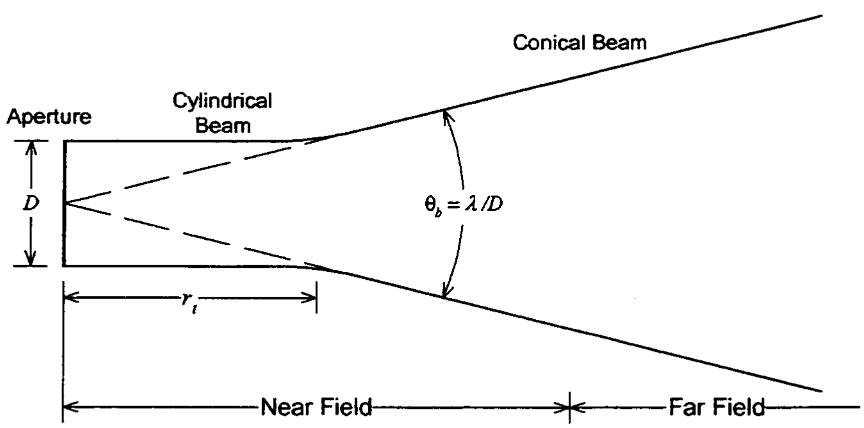

Figure 5: Open-ended waveguide and field regions

By the geometry shown,

$$\tan\left(\frac{\theta_b}{2}\right) = \frac{D/2}{r_t}. \tag{25}$$

For very narrow beams (large apertures), $\tan(\theta_b/2) \approx \theta_b/2$. Using $\theta_b = \lambda/D$, the transition point is

$$r_t = \frac{D^2}{\lambda}. \tag{26}$$

The generally accepted point of the start of the far field is twice the distance from the phase centre to this point. Hence,

$$r_{ff} = \frac{2D^2}{\lambda} \tag{27}$$

which we note is identical to an earlier definition of far field derived for linear wire antennas.