Atmospheric Effects

The earth's atmosphere has characteristics that affect the propagation of radio waves. These effects happen at different points in the atmosphere, and hence it is worth discussing the structure of the earth's atmosphere briefly.

The earth's atmosphere can be divided into the following regions:

- *Troposphere* – Existing from the earth's surface to between 7-17 km above the earth's surface, this region of the atmosphere is the most turbulent since there is a lot of thermal movement of air. Air temperature decreases strongly as altitude increases due to expansive cooling: as altitude increases, gases can expand due to the reduction of pressure which reduces the temperature of the gas.
- *Stratosphere* – Starts at the troposphere and ends at an altitude of about 50 km. The ozone layer is located here.
- *Mesosphere* – Existing between 50 to 80-85 km above the Earth's surface.
- *Thermosphere* – Located beyond the mesosphere, it extends to 640+ km and contains the ionosphere which is a region of highly charged particles which results from the ionization of atmosphere atoms/molecules from solar radiation. The ionosphere plays a major role in radio wave propagation below 40 MHz.
- *Exosphere* – The remainder of the atmosphere past the mesosphere, extending to approximately 10,000 km.

The most important atmospheric effects on radio wave propagation are *refraction* and *reflection*. Refraction can occur in the troposphere or the ionosphere. Tropospheric refraction occurs because the refractive index of the atmosphere decreases as altitude increases, leading to a bending of waves back toward the earth. Conversely, ionospheric refraction occurs because of the electrical properties of plasmas that are formed in the ionosphere as a result of ionization of the atmosphere. Reflection off the ionosphere is also possible if the frequency is low enough. We will differentiate these two effects, and refer to the former as *atmospheric refraction* and the latter as *ionospheric propagation*. The atmosphere will also *attenuate* radio signals, due to absorption by air molecules, water molecules, and precipitation (rain).

Atmospheric Refraction

Refraction in the troposphere produces rays, initially launched horizontally (as in the case of the radio links we have discussed already), bending along with the curvature of the earth. To understand this process, we first examine the refractive index of the atmosphere as a function of altitude. As we know, the refractive index of air is very close to one, and since it is so close, the *difference* between the refractive index and that in a vacuum is of most interest ($n - 1$, where $n = \sqrt{\varepsilon_r}$ denotes the refractive index of the atmosphere having dielectric constant ε_r).

Since $n-1$ is very small, it is useful to define *refractivity* which is equal to

$$N = 10^6(n-1) \tag{1}$$

For example, a typical value of the refractive index at the Earth's surface is $n_0 = 1.000350$, which is equivalent to $N = 350$ N-units. The refractivity of the atmosphere is a function of many things, including the air pressure (p), the absolute temperate (T), and the humidity (e). A commonly used expression is

$$N = \frac{77.6}{T}\left(p + \frac{4810e}{T}\right) \tag{2}$$

where p is in millibars, T is in Kelvin, and e is expressed as the partial pressure of water vapour, in millibars. The most important thing to note is that refractivity is inversely proportional to temperature and directly proportional to pressure and humidity. Hence, as we go higher into the atmosphere, the refractivity tends to drop somce the pressure is less, and the air is more dry. Temperature plays a role as well, and in reality temperature gradients can cause the refractivity profile to be non-monotonic.

The refractivity of the atmosphere decreases as you get higher into the atmosphere. This leads to a curved propagation path for rays that are incident to the atmosphere at an angle. To see how the bending into a curve happens, we can begin by treating the atmosphere as stratified medium, represented by many small planar layers each containing a different refractive index that changes with altitude. This concept is shown in Figure 1.

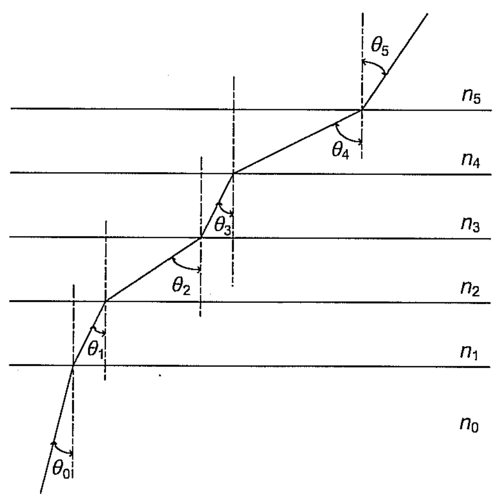


Figure 1: Stratified media representation of atmosphere

Then, from Snell's Law,

$$n_0 \sin\theta_0 = n_1 \sin\theta_1 = n_2 \sin\theta_2 = \cdots = n_n \sin\theta_n = \cdots \tag{3}$$

which shows that the angle of refraction increases for $n_1 < n_2 < \cdots\ n_n < \cdots$. Now, as we let the layers become infinitely thin and infinite in number, we can think of n and θ becoming continuous functions of h such that

$$n(h)\sin\theta(h) = n_0 \sin\theta_0 \tag{4}$$

Now we consider the more accurate situation of a spherical earth with spherically stratified layers. One such layer is shown in the figure below. Consider a bundle of rays propagating through a material with a refractive index gradient discussed earlier, as shown in Figure 2. The rays are curved such that their radius of curvature is ρ. All rays in the bundle must traverse the material in the same time, dt, if they are considered to be part of the same bundle. Here, we consider two rays in that bundle.

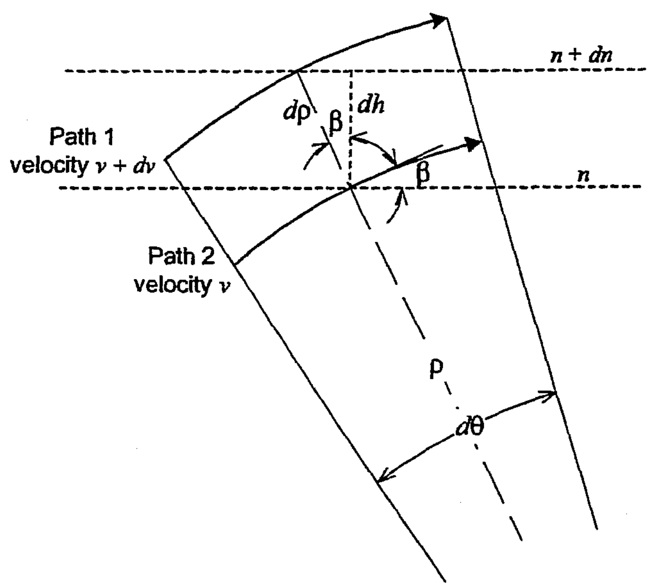


Figure 2: Ray bundle traversing the atmosphere

Consider path 2 first. The wave traversing that path, which has a radius of curvature ρ, has a velocity v and traverses the path in a time dt. Therefore, we can write

$$\rho d\theta = v dt \tag{5}$$

Now consider path 1. The refractive index is different since it is higher in the atmosphere, so we shall denote the refractive index at $n + dn$ (although we realize that dn will be negative from what we know of the atmosphere), and the associated velocity $v + dv$. Therefore,

$$(\rho + d\rho)d\theta = (v + dv)dt \tag{6}$$

Dividing equations (6) and (5), we obtain

$$1 + \frac{d\rho}{\rho} = 1 + \frac{dv}{v} \tag{7}$$

from which it follows that

$$\frac{d\rho}{\rho} = \frac{dv}{v}. \tag{8}$$

We also know that $nv = c$, or $n = c/v$. Differentiating,

$$dn = -\frac{c}{v^2}dv = -\frac{n}{v}dv \Rightarrow \frac{dn}{n} = -\frac{dv}{v} \tag{9}$$

It then follows from equations (8) and (9) that

$$\frac{dn}{n} = -\frac{d\rho}{\rho} \Rightarrow \frac{1}{\rho} = -\frac{1}{n}\frac{dn}{d\rho} \tag{10}$$

The inverse of a radius of curvature is called *curvature*. Hence, the relation describes the curvature of the ray; if we know $dn/d\rho$, then the amount of ray bending can be found.

From the illustration, if the angle β is small (the rays enter close to parallel to the atmospheric layer), we can say that $d\rho \approx dh$, and since n is close to one, the curvature can be expressed as

$$\frac{1}{\rho} \approx -\frac{dn}{dh}. \tag{11}$$

So, if we know the refractive index profile as a function of height (dn/dh), we can find the curvature of the path. Notice what happens if there is no change in refractive index ($dh/dn = 0$): the curvature is zero (or the radius of curvature is infinity), and there is no bending of the ray – it is a straight line.

We are interested in the difference between the path curvature and the curvature of the earth. If the radius of the earth is R_e, then the curvature of the earth is $1/R_e$. The difference in curvatures is

$$\frac{1}{R_e} - \frac{1}{\rho} = \frac{1}{R_e} + \frac{dn}{dh}. \tag{12}$$

Notice that if there is no refractive index gradient ($dn/dh = 0$, then $\rho \to \infty$ meaning that the ray is not bent. But if $dn/dh \neq 0$, we can produce curvatures that are less than or greater than the curvature of the earth (the former is much more common).

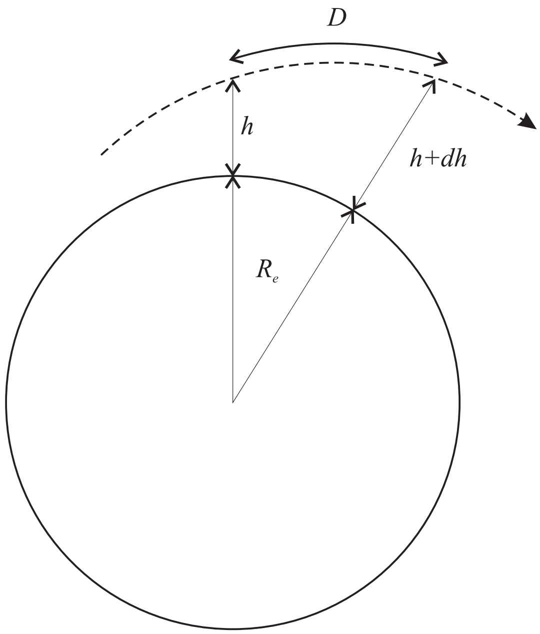
The validity of equation (12) is unchanged if we set it equal we set the value of it to $1/KR_e$. This is equivalent to enlarging the earth radius by a factor of K and replacing the curved path with an equivalent straight path (since a path with zero curvature is a straight line). The geometry of the two situations is the same, as shown in Figure 3. Notice how in the diagram, the straightening of the path increases the horizontal distance between two intersection points on the path, requiring an increase in radius to keep h and $d + dh$ constant.

Therefore, we have

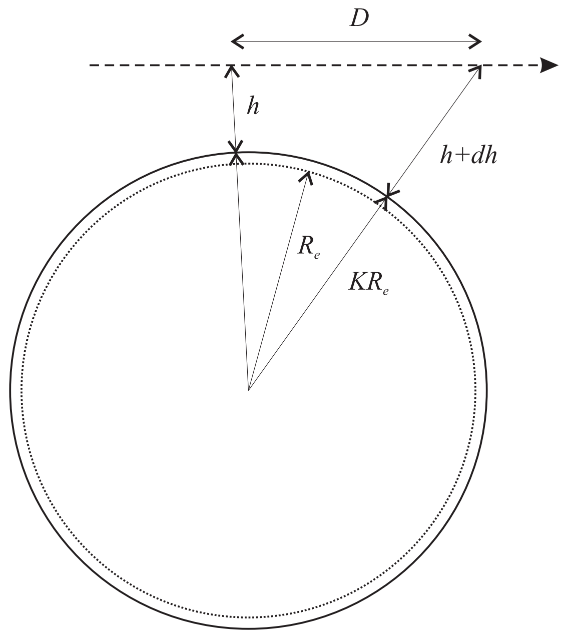
$$\frac{1}{KR_e} = \frac{1}{R_e} + \frac{dn}{dh} \tag{13}$$

or

$$\frac{1}{K} = 1 + R_e\frac{dn}{dh} = 1 + 10^{-6}R_e\frac{dN}{dh}. \tag{14}$$



(a) Original geometry



(b) Equivalent geometry

Figure 3: Propagation scenarios

A slightly more accurate version uses (11) instead of the approximation (10), in which case we have

$$\frac{1}{K} = 1 + \frac{R_e}{n_0}\frac{dn}{dh} = 1 + 10^{-6}\frac{R_e}{n_0}\frac{dN}{dh}. \tag{15}$$

This equation can be evaluated by assuming a form for $n(h)$ and using experimentally measured refractivities at different altitudes. Solving this equation gives K values between 1.2 to 2.1 in North America. A common value is $K = 4/3$. Hence, the radius of curvature of the path is 1.2-2.1 times larger than that of the radius of the earth, leading to much longer propagation distances than if the path was straight.

To see this, consider a tower of height h that broadcasts a signal so that it is "horizontal"; i.e., perpendicular to the tower. The maximum distance d that the transmitter can broadcast is to the horizon, as shown in Figure 4. For the geometry,

$$(a + h)^2 = a^2 + d^2 \tag{16}$$

For small h, we can disregard the h^2 term and

$$d = \sqrt{2ah} \tag{17}$$

If there was no atmospheric refraction, we would simply use $a = R_e$ in the formula, But, when atmospheric effects are taken into account, the waves travel with an effective radius that is not $a = R_e$, but $a = KR_e = (4/3)R_e$. Hence, propagation is possible over a large distance because the rays curve to follow the contour of the earth.

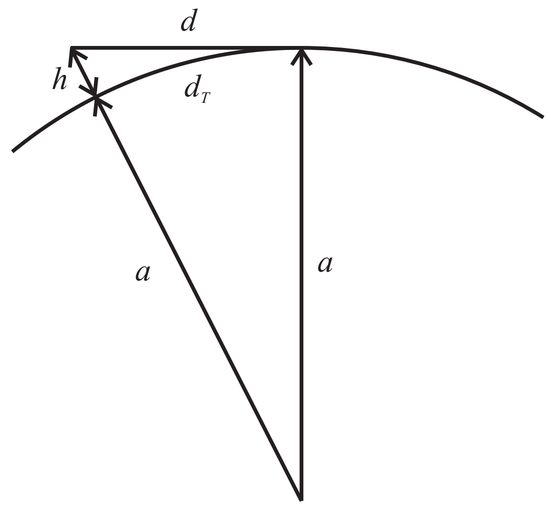


Figure 4: Link geometry

Example: For $R_e = 6368$ km and a tower with height $h = 100$ m, the maximum distance without refraction (the LOS distance) is $d = 37.56$ km. With $K = 4/3$, the distance becomes $d = 41.2$ km.

The nonlinear relationship between K and dN/dh is shown in Figure 5. The *normal region* between 0 to -79 N-units/km produce K values between 1 and 2. The plot also shows that under certain conditions, it is actually possible to have some very exotic K values. For $-159 \leq N \leq -79$ N-units, this is called *super-refractivity* because K values can range from 2 to ∞. At $dN/dh = -157$ N-units/km, the curvature of the ray matches the curvature of the earth and the ray maintains the same altitude as it travels. As dN/dh takes on increasingly negative values, which is known as a *trapping gradient*, the curvature of the rays is actually smaller than that of the curvature of the earth, meaning that the rays return to the earth.

Each of these situations is summarized in terms of the effective earth radius and reality in the plots shown in Figure 6. The case shown in the top sub-plot is the most common, and produces a curvature less than that of the Earth. This means that if the angle of the transmitter ψ_0 is chosen strategically, that beyond-the-horizon communication is possible as illustrated numerically in the example earlier. Note that for the rays to curve and return to earth, requires that at some point that total internal reflection occurs at one of the layers so that the refraction process reverses itself as the ray returns back to earth (the curvature remains unchanged). Hence, refractive index profiles that are monotonically decreasing will not allow rays to return to earth; there must be a region of the atmosphere where dN/dh is positive to produce total internal reflection. Given the complex dependence on pressure, temperature, and humidity, such profiles are achievable in practice.

The middle subplot shows the case when $dN/dh = -157$ N-units/km, which produces $K = \pm\infty$. At this point the curvature of the rays is exactly equal to the curvature of the earth, so they remain parallel to the earth at the same altitude above the sruface with no apparent bending. Hence the effective earth radius model shows a planar earth with no curvature.

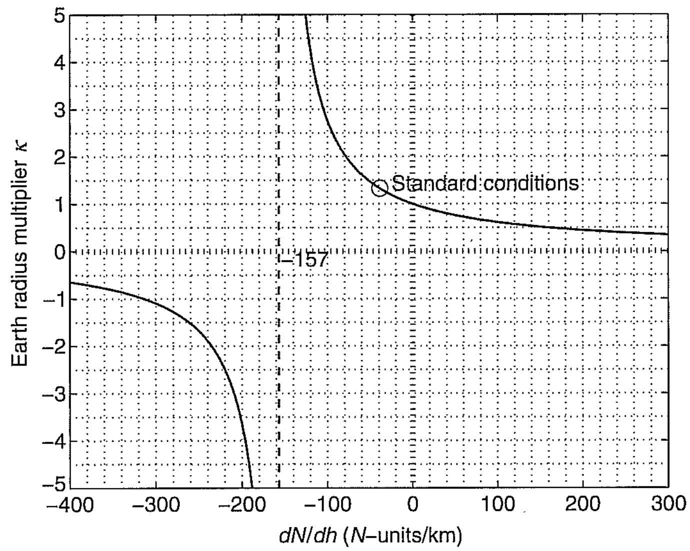


Figure 5: Plot of earth radius multiplier versus refractive index gradient

Finally, the bottom subplot shows the super-refractive case which produce curvatures that return the ray to earth much sooner than in the case of no refraction. The effect of this is to invert the curvature of the earth, in the effective Earth radius model.

Figure 7 shows the computed ray trajectories for an atmospheric refractivity profile shown in the top subplot. The elevation angle ψ_0 of the transmitter is varied in order to change the effective range of the radio link. Notice that for small angles aimed slightly over the horizon $(\psi_0 = 0°)$ it is possible to extend the range of the radio link. Here, plane-earth reflection has been taken into account so multiple bounces of the radio signal are shown. However, beyond a certain elevation angle, the signal never returns to earth since the ray angles are too large to produce a total internal reflection condition at one point during the refraction process. These rays never return to earth. As the elevation angle continues to increase, the curve becomes less affected by refraction; certainly by the time ψ_0 is a few degrees, the ray trajectories are essentially straight.

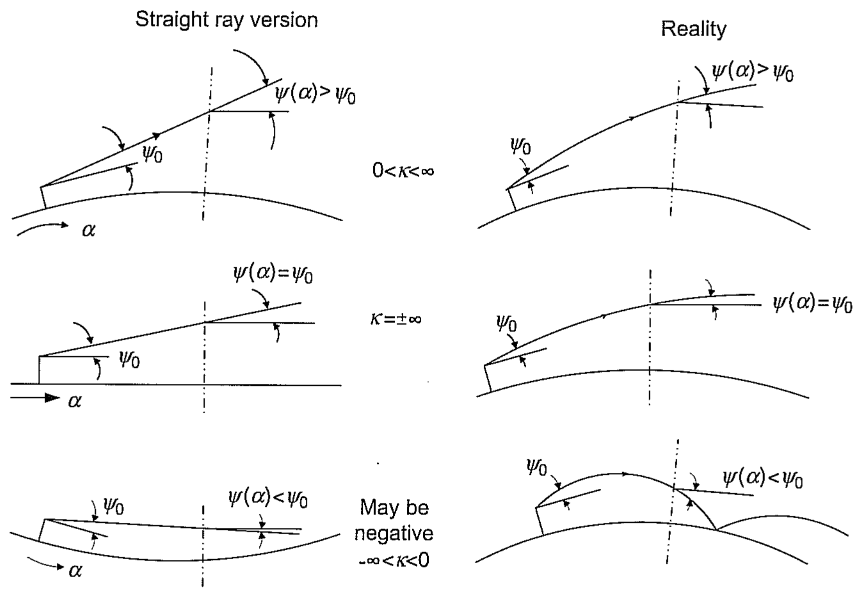


Figure 6: Refraction cases

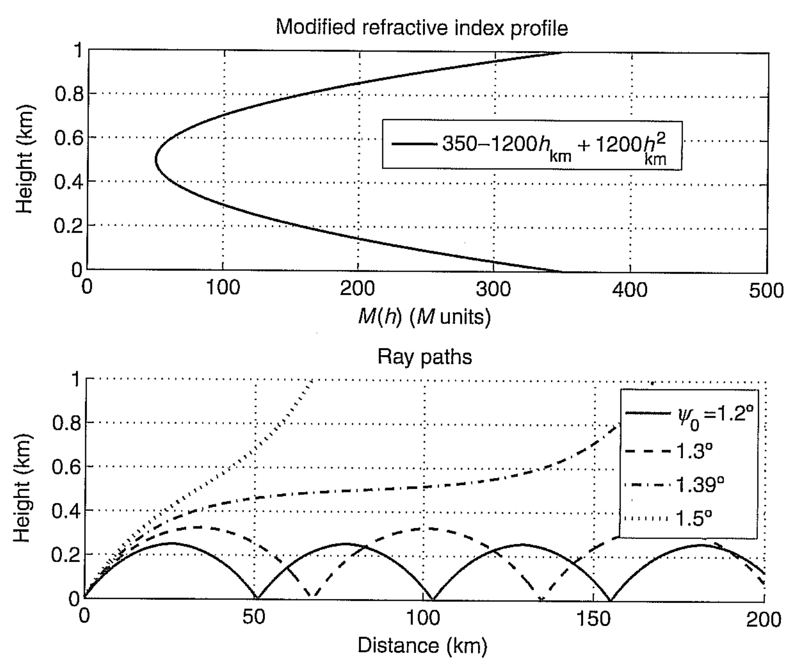


Figure 7: Ray-tracing calculations of ray trajectories for given atmospheric refractivity gradiant