Radar Systems

Radar stands for RAdio Detection And Ranging. It is a type of radio system where radio signals are used to determine the position or speed of an object. Often, the object is passive, so the reflection of the radio signal off of the object is used to determine the speed or velocity of the object. Radar is used for a variety of purposes, including weather monitoring, air traffic control, speed enforcement, astrometry, and military applications.

1 Radar Range

Radar signals are usually pulses that are modulated onto an RF carrier. Pulses are used so that multiple objects can be resolved in the time domain. In the simplest case, for a single reflector, a pulse with measured round-trip time t allows the range to the object to be calculated as

$$R = \frac{ct}{2}$$

The *range resolution*, ΔR, is

$$\Delta R = \frac{c\tau}{2}$$

where τ is the pulse width. If two targets, shown graphically in Figure 1, are closer than $\Delta R = R_A - R_B$ apart, the pulses overlap and the two objects cannot be distinguished. There is usually a compromise between range resolution and practical limitations of an RF system: narrower pulses consume more bandwidth, and are more difficult to detect unless special signal processing techniques are employed.

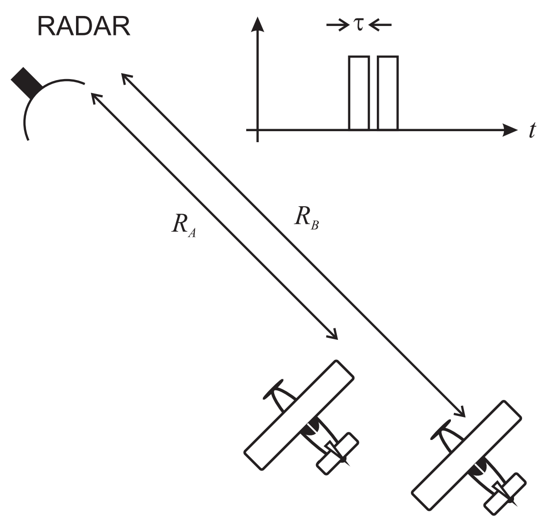


Figure 1: RADAR problem with two targets

Pulses are usually transmitted periodically so that range information can be provided in real time. The pulse repetition frequency (PRF) is the rate of transmit pulse repetitions:

$$f_r = \frac{1}{t_r}$$

Introducing periodic pulses constrains the radar system as well, since if a target is located beyond a range

$$R_u = \frac{ct_r}{2}$$

then the received pulse arrives after the next pulse has already been transmitted, resulting an ambiguity as to which transmit pulse the received pulse was associated with in the first place.

To resolve the direction of a target, high gain antennas are used which have very high angular resolution. By mounting the antenna on a rotating platform, the antenna can *scan* an area for targets, producing radar data in angular sweeps. The type of beam varies with the application. For interferometry and air traffic control, a *fan beam* is usually employed which is very narrow in the azimuth direction but quite wide in the elevation plane. How would one shape an aperture antenna to produce such a beam?

2 Types of Radar Systems

Radar systems can be implemented in one of two ways, which are summarized in Figure 2. In a *monostatic* radar system, the transmitter and receiver are co-located at the same physical location, as shown in diagram (b). The transmitter and receiver may even share the same antenna if a *duplexer* is used to switch the antenna between the transmitter and receiver at the right points in time. Alternatively, a *circulator* can be used (as shown in diagram (a)) which only allows power flow from port to port in the direction shown by the arrow (it is an example of a *non-reciprocal network*).

In some applications, the receiver is located at some distance remote from the transmitter. This arrangement is called a *bistatic* radar system, as shown in diagram (c). It is more common in weather measurements or in military applications.

3 Radar Reflection

Let's analyze a *monostatic* radar system where the same path is traversed by a radio signal to and from the target. To determine the power of the received signal, we simply use a link budget as we would for any other radio system. There are some subtle differences, since the target essentially acts as a re-radiator. The power density incident on the target at a distance R is given by

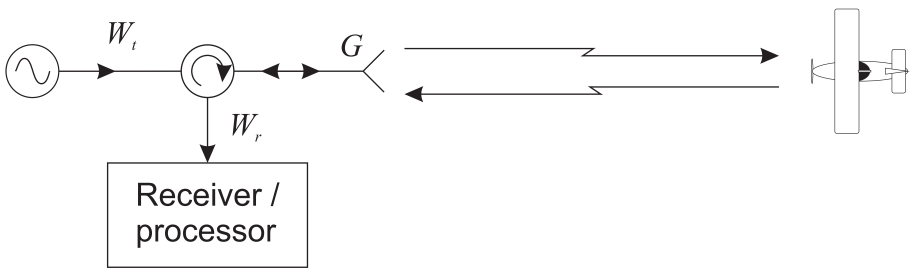
$$P_i = \frac{W_t G_t}{4\pi R^2 l}$$

where losses in addition to free-space loss are lumped together in a single loss term l. The amount of power reflected by a passive target is known as the target's *radar cross section*, σ, which obviously must have units of area. Radar cross section is defined as

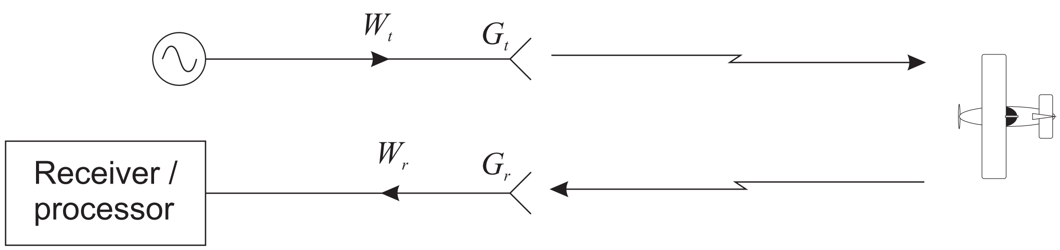
$$\sigma = 4\pi R^2 \frac{P_r}{P_i} \tag{1}$$

and is somewhat analogous to effective area for antennas. The resulting power density at the receiver is given by

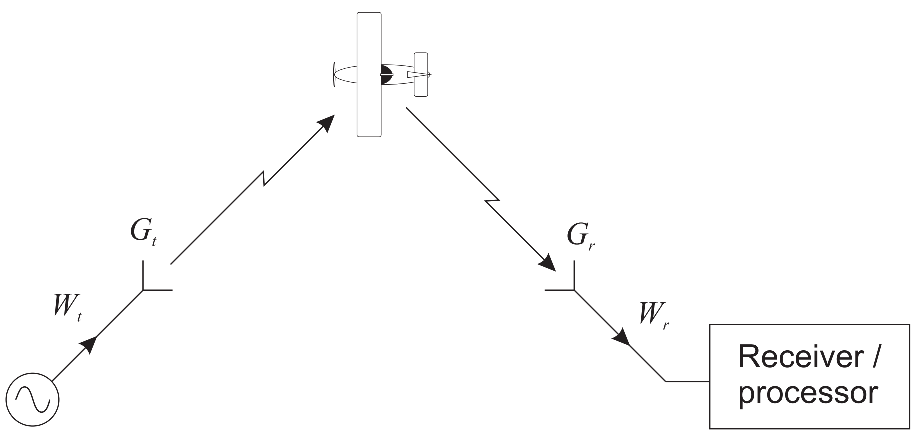
$$P_r = \frac{P_i \sigma}{4\pi R^2 l} = \frac{W_t G_t \sigma}{(4\pi R^2 l)^2}$$



(a) Monostatic radar with single antenna



(b) Monostatic radar with two antennas



(c) Bistatic radar

Figure 2: Types of RADAR systems

The received power at the receiver is then equal to

$$W_r = A_{eff,r} P_r = \frac{A_{eff,r} W_t G_t \sigma}{(4\pi R^2 l)^2}$$

Notice how the received power is inversely proportional to R^4, due to the round-trip nature of the signal path.

As we learnt when we studied noise systems, ultimately the sensitivity of the receiver is limited by the noise power in the system. This will thus impose a minimum received power limit on the radar system which establishes the maximum useful range for the radar. If this minimum useful signal power is W_{min}, then the maximum useful range of the radar is

$$R_{max} = \left[\frac{A_{eff,r} W_t G_t \sigma}{(4\pi l)^2 W_{min}} \right]^{1/4}$$

The radar cross section of the target, therefore, plays a very large role in how easily a target is detected at a certain distance. A table of RCS values for various objects is shown in Table 1. Note that σ can also be expressed in dBsm (dB relative to a square meter, or $10 \log \sigma$).

Table 1: RCS values of various objects

Target	σ (m^2)
Insect	10^{-4}
Bird	0.01
Missile	0.5
Person	1
Fighter jet	5–100
Airliner	100–1000
Ship	3,000 – 1,000,000
B-2 stealth bomber	10^{-6} – 10^{-4}

Sometimes, the return signals from a radar signal are larger than that from a passive reflector. This is often the case when the radar target *wants* to be found, such as in air traffic control. A transponder on the aircraft generates the "reflection", and often encodes additional information with the transmission.

If the EIRP of the transponder/beacon is $EIRP_b = W_b G_b$, then the received power is

$$W_r = \frac{A_{eff,r} EIRP_b}{4\pi R^2 l} = \frac{A_{eff,r} W_b G_b}{4\pi R^2 l}.$$

The maximum range for an active transponder can then be found as

$$R_{max} = \left[\frac{A_{eff,r} W_b G_b}{(4\pi l) W_{min}} \right]^{1/2}$$

Note that effective area of the receiver antenna is chosen to be that used in the radar system; in the case of a monostatic radar using the same antenna for transmitting and receiving, the effective area of the antenna is obviously the same as that of the transmitting antenna.

A bistatic RADAR works similar to a monostatic radar, except that the link from the transmitter to the target is different from the target to the receiver. The incident power density is

$$P_i = \frac{W_t G_t}{4\pi R_1^2 l_1},$$

where R_1 is the distance from the transmitter to the target, and l_1 are the losses along the same path. Then,

$$W_r = A_{eff,r} P_r = \frac{A_{eff,r} W_t G_t \sigma}{(4\pi)^2 R_1^2 R_2^2 l_1 l_2}$$

where R_2 and l_2 are associated with the path between the receiver and the target.

4 Radar Cross Section of Some Simple Targets

We will consider scattering from two simple geometrical targets: a sphere, and a flat plate.

4.1 RCS of a Sphere

Consider a perfectly conducting sphere, of radius a, as shown in Figure 3.

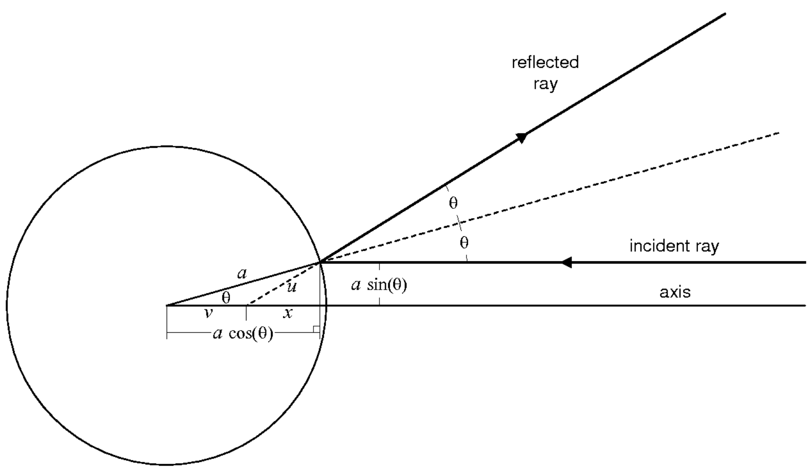


Figure 3: Electromagnetic wave incident upon a conducting sphere

An incident ray from a plane wave impinges upon the sphere, incident at an angle θ normal to the sphere. The wave produces a reflection leaving the sphere, that makes the same angle with the normal. If we extend the reflected ray back into the sphere, it appears to emanate from a point a distance v away from the sphere's centre, along the axis of incidence. We are interested in finding this distance v.

Analyzing the triangle formed by the axis of incidence and the reflected ray, we find that

$$\cot(2\theta) = \frac{x}{a \sin\theta}$$

It then follows that

$$\begin{aligned} v &= a\cos\theta - x \\ &= a\cos\theta - a\sin\theta\cot(2\theta) \\ &= a\cos\theta - a\cancel{\sin\theta}\frac{(1-\tan^2\theta)\cos\theta}{2\cancel{\sin\theta}} \\ &= a\cos\theta\left(1 - \frac{1}{2} + \frac{1}{2}\tan^2\theta\right) \\ &= a\cos\theta\left(\frac{1}{2} + \frac{1}{2}\tan^2\theta\right) \end{aligned}$$

where the trigonometric identity $\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$ was used. For small angles θ, the rays leaving the sphere are approximately paraxial, and

$$v \approx \frac{a}{2}$$

which means that these rays appear to originate from a point a distance v away from the centre of the sphere along the axis of the sphere, as shown in Figure 4. We can treat this point as a source of spherical waves.

Suppose the incident power density at the sphere is P_i. This power is reflected by the sphere and appears to originate from a point as described above. Hence, if we choose an area A_i on the surface of the sphere which is considered to be generating the reflected power density, then at a distance R from the sphere, conservation of power requires that

$$P_i A_i = P_r A_r$$

or

$$\frac{P_r}{P_i} = \frac{A_i}{A_r}.$$

The ratio of the two areas is easily found using geometry to be

$$\frac{A_i}{A_r} = \frac{(a/2)^2}{R^2}.$$

Therefore,

$$\sigma = 4\pi R^2 \frac{P_r}{P_i} = \pi a^2$$

which is an amazingly simple result that is independent of frequency. For this reason, spheres are often used as calibration standards for RCS measurement systems. The result is easy to remember as it is the cross-sectional area of the sphere. This result holds for spheres much larger than a wavelength in radius.

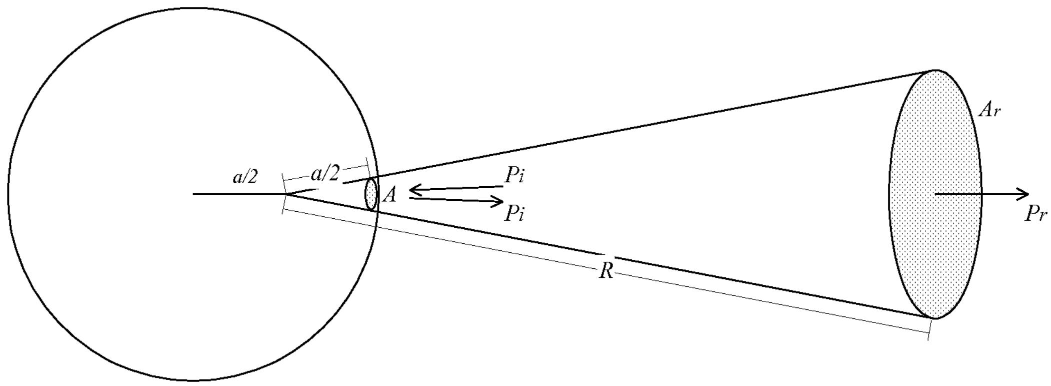


Figure 4: Scattering from a sphere

4.2 RCS of a Flat Plate

The reflected power density from a sufficiently large (with respect to a wavelength) flat perfectly conducting object can be easily computed by considering the reflector to be an aperture antenna with effective area A. The reflected power emanating from the plate is then

$$W_p = P_i A$$

and we can view the plate has having a "gain"

$$G_p = \frac{4\pi}{\lambda^2} A$$

which results in a reflected EIRP of $W_p G_p$. The received power density is then

$$P_r = \frac{W_p G_p}{4\pi R^2}$$

from which it follows that

$$\sigma = 4\pi R^2 \frac{P_r}{P_i} = \frac{4\pi A^2}{\lambda^2}.$$

Objects having a measured RCS of σ_0 can be seen has having equivalent plate areas using this relationship:

$$A_{eq} = \frac{\lambda}{2}\sqrt{\frac{\sigma_0}{\pi}}$$

5 Radar Systems

5.1 Example Pulsed Radar System

Figure 5 illustrates a practical implementation of a monostatic pulsed radar system. The "pulse" in this case consists of a frequency shifted carrier which is generated by mixing the local oscillator signal at frequency f_{LO} with an intermediate frequency (IF) signal at frequency f_{IF} in the transmitter. This signal is transmitted periodically when the transmit-receive switch to the antenna connects the transmitter to the antenna. The pulse generator is configured to create pulses of width τ, repeating every T_r ($= 1/\text{PRF}$) seconds.

When the switch is in the other position, the receiver is connected to the antenna. In this way, the transmit/receive switch connected to the antenna, in concert with the pulse generator, perform a *duplexing* operation to switch the mode of the system. In receive mode, the received signal is amplified and downconverted to baseband. Any IF component in the downconverted signal is detected by the detector, amplified, and output to a display for processing.

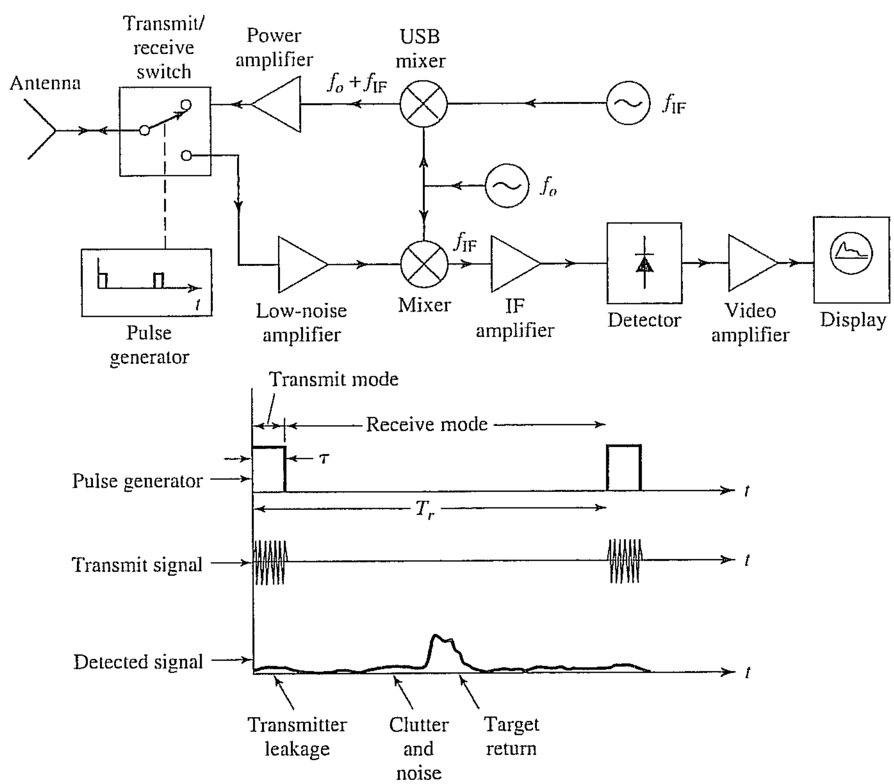


Figure 5: Pulsed radar system

5.2 Doppler Radar

Doppler radar refers to radar systems designed where the transmitter and/or receiver are in motion. The well known Doppler effect produces a perceived up-shift or down-shift in frequency when a constant frequency source is moved toward or away from an observer, respectively. The exact *Doppler shift* in frequency can be calculated by considering Figure 6.

Consider a perfect conducting plate is moving at a velocity v in the $+z$ direction. A plane wave with frequency $\omega = 2\pi f$ is also travelling in the $+z$ direction towards the plate. This plane wave

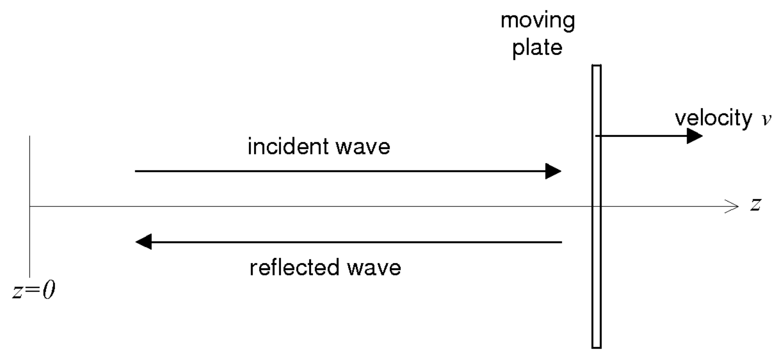


Figure 6: Incident and reflected waves from a moving PEC plate

is given by

$$E_i = E_0 e^{j\omega(t-z/c)} = E_0 e^{j\omega(1-v/c)t}$$

A reflection is generated at the conductor surface which has reflection coefficient Γ. This wave is perceived as having a slightly different frequency ω':

$$E_r = \Gamma E_0 e^{j\omega'(t+z/c)} = \Gamma E_0 e^{j\omega'(1+v/c)t}$$

At the surface of the conductor, the tangential fields must be zero. Assuming the incident field is polarized parallel to the conductor,

$$E_i + \Gamma E_r = 0$$

and since $\Gamma = -1$ for a PEC,

$$\begin{aligned} E_i &= E_r \\ e^{j\omega(1-v/c)t} &= e^{j\omega'(1+v/c)t} \\ \omega(1-v/c) &= \omega'(1+v/c). \end{aligned}$$

The difference between the perceived and actual frequency can be expressed as

$$\omega' - \omega = -(\omega' + \omega)\frac{v}{c} = \Delta\omega$$

The carrier frequency ω is usually much, much larger than the produced Doppler shift. Therefore, $\omega' + \omega \approx 2\omega$, and

$$\Delta\omega = -2\omega\frac{v}{c}$$

or

$$\Delta f = f_d = -2v\frac{f}{c} = -2\frac{v}{\lambda}.$$

Notice that the Doppler frequency in this case is negative, meaning that the perceived frequency at the receiver is lower than the actual frequency of the transmitter.

If the plate does not move directly away from the observer at $z = 0$, but rather along a linear path at an angle θ with the z-axis, then

$$f_d = -2\frac{v}{\lambda}\cos\theta.$$

Doppler radar is used in many systems for detecting the velocity of objects. Speed enforcement and military applications are the most obvious examples. Doppler radar is also used to track storms, since the movement of a storm cell produces measurable Doppler effects that can be used to plot the course of the storm using Radar.

An example of an implementation of a monostatic Doppler radar system is shown in Figure 7. A circulator is used to isolate the transmitted and received signals in the system. Direct mixing of the transmitted and received signal yields the Doppler frequency at the IF output of the mixer, which can be subsequently filtered, amplified, and displayed.

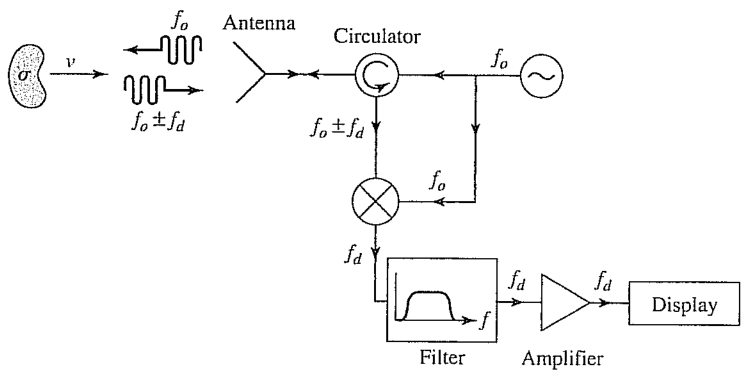


Figure 7: Doppler radar system

5.3 FM CW Radar

A radar system that uses a continuous wave (CW) signal, versus a pulse, is known as a CW radar system. To detect the distance and/or velocity of objects, the frequency of the transmitted signal is swept (modulated) with time, hence the FM name. The frequency is usually swept linearly with time; such a signal is sometime called a "chirp" though the definition is quite broad.

A plot of the frequency of the transmitted signal versus time is shown in Figure 8. At the end of the up-shift, the frequency of the signal is ramped back down, though this is optional and needed only if there are two unknowns in the system (position and velocity). Shown in the dashed line is the received signal (echo) frequency, which is obviously time-delayed relative to the transmitted signal.

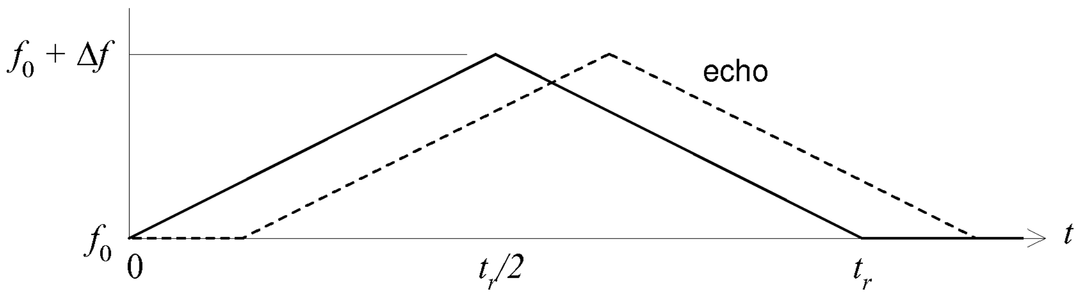


Figure 8: FMCW radar system

The instantaneous transmitted waveform frequency can be expressed as

$$\begin{aligned} f(t) &= f_0 + 2f_r\Delta f t, & t \leq t_r/2 \\ f(t) &= f_0 + \Delta f - 2f_r\Delta f(t - t_r/2), & t > t_r/2 \end{aligned}$$

where $f_r = 1/t_r$ is the frequency of the ramp signal. The frequency of the received echo is simply a time-shifted version of this function, plus any Doppler shift added due to movement of the target:

$$f_e(t) = f(t - 2R/c) + f_d$$

If these two signals are applied to the inputs of a mixer, the *difference frequency signal* can be used to estimate the range and speed of the target. Where the transmitted and received signals overlap and have positive slopes (the "up-frequency" segment), the frequency difference is

$$f_+ = 4Rf_r\Delta f/c - f_d$$

while on the "down-frequency" segment, the difference is

$$f_- = -4Rf_r\Delta f/c - f_d.$$

Hence, by observing the mixer output at both stages of the frequency sweep, enough data can be obtained to solve these equations simultaneously, leading to

$$R = \frac{c(f_+ - f_-)}{8f_r\Delta f}$$

and

$$f_d = -\frac{f_+ + f_-}{2}.$$

The latter equation can be used to solve for the target's velocity.