Passive Delay-Based Macromodels for Signal Integrity Verification of Multi-Chip Links

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Abstract
This paper presents a general strategy for the electrical performance assessment of electrically long multi-chip links. A black-box time-domain macromodel is first derived from tabulated frequency responses in scattering form. This model is structured as a combination of ideal delay terms with frequency-dependent rational coefficients. Two passivity enforcement schemes are then presented, enabling safe and reliable transient simulations of the multi-chip link with a standard circuit solver, including nonlinear drivers and receivers.

Introduction
This paper is about electrical modeling of high-performance chip-to-chip interconnects for Signal Integrity verifications. Two main challenges must be faced in the numerical simulation of such structures. First, the electrical length of the link may be considerable, imposing a strong requirement on electrical models, that must take into account the distributed nature of the link and the inevitable propagation delays. Second, the signals may undergo several discontinuities along their propagation, coming from vias, connectors, or return path discontinuities due to routing constraints or irregular power/ground planes at the package and board level. Spurious signal reflections may arise, which must be correctly represented in electrical models.

A standard modeling approach is to characterize the entire link in the frequency domain, in order to accurately represent all signal degradation effects, including frequency-dependent metal and dielectric losses. Each elementary block forming the interconnect is analyzed separately via full-wave tools (2D for pure transmission-line segments and 3D for via fields, connectors and lumped discontinuities). The overall response of the entire chip-to-chip link is obtained by cascading the different blocks at each individual frequency in the range of interest, leading to a set of tabulated frequency responses in scattering form. The same description may be obtained, when feasible, via direct measurement using a multiport Vector Network Analyzer (VNA).

Frequency-domain analysis is straightforward once the above information is available. Standard FFT methods may then be used to recover the time-domain impulse or step responses, which in turn may be processed to derive global metrics such as eye diagram openings or Bit Error Rate (BER) statistics. The main drawback of this approach is in the intrinsic assumption of linearity. Nonlinear models for drivers and receivers cannot be used, thus limiting the scope and the representativeness of the results.

This work presents a different approach. Building on various existing results, a time-domain macromodeling procedure is presented. The raw tabulated frequency responses are fed to a delay-rational identification scheme, which produces a parametric closed-form model. This model is successively processed by a passivity check and enforcement procedure and synthesized as a SPICE-compatible netlist. Due to the passivity constraint, this model can be safely employed in transient analysis, including accurate nonlinear models of drivers and receivers. The main contributions of this work are two alternative passivity enforcement schemes, both including an accuracy preservation methodology based on suitable weighted norms.

Delay-Rational Macromodels
We consider a generic chip-to-chip link with \( P \) electrical ports. The structure is known via its sampled scattering matrix \( \tilde{S}_l \in \mathbb{C}^{P \times P} \) at the discrete frequencies \( \omega_l, \ l = 1, \ldots, L \). A Delay-Rational Macromodel (DRM) is defined as

\[
S(s) = \sum_{m=1}^{M} Q_m(s) e^{-s \tau_m},
\]

where \( s \) is the Laplace variable, \( \tau_m \) are delays corresponding to the various arrival times of the signal reflections induced by an input unit pulse, and \( Q_m(s) \) are matrix rational coefficients representing other effects such as attenuation and dispersion. The case \( M = 1, \tau_1 = 0 \) corresponds to a standard (delayless) purely rational macromodel. Rational macromodeling has received a lot of attention in the last decade and is now a standard practice [1]. This approach is unfortunately not applicable to electrically-long chip-to-chip links without facing serious complexity issues, as discussed in [2]-[5].

The identification of (1) from the samples \( \tilde{S}_l \), i.e., solving

\[
\min ||S(j \omega_l) - \tilde{S}_l||,
\]

where the minimum is taken over the unknown delays \( \tau_m \) and matrix rational functions \( Q_m(s) \) is a very challenging task. However, good solutions via Delayed Vector Fitting (DVF) or Delayed Sanathanan-Koerner (DSK) iterations are available, see [4]. For this reason, no further details will be provided here about the actual identification of the macromodel (1), and the rest of this paper will focus on its passivity characterization and enforcement. To this end, we recall that a delayed state-space realization

\[
S(s) = \left( \sum_{m=1}^{M} C_m e^{-s \tau_m} \right) (sI - A)^{-1} B + D
\]

may be derived from (1) using standard methods. This will be our starting point in next section.

Passivity characterization and enforcement
Electrical interconnects are physically passive, i.e., they are unable to generate energy. Mathematical models in form (1) or (3) may however lose this property due to numerical approximations in the identification process. Unfortunately, non-passive macromodels may lead to unstable results when used...
in transient simulations, even when their terminations are passive [6, 7]. Therefore, the model must be checked for passivity and, if passivity is violated within some frequency bands, a suitable passivity enforcement process must be applied.

We assume that all poles of all matrix rational coefficients $Q_m(s)$ have a strictly negative real part. This is enforced by any standard rational identification scheme such as VF, DVF, or DSK [4]. In addition, we assume that $Q_m(s)$ is strictly proper whenever the associated delay $\tau_m \neq 0$. Under these assumptions, it can be shown [8] that the model (3) is passive when

$$ (I - S(j\omega)S(j\omega)) \geq 0, \quad \forall \omega $$

or, equivalently, when all the singular values of $S(j\omega)$ do not exceed one at any frequency.

A direct check of (4) requires the evaluation of the singular values of $S(j\omega)$ for many values of $\omega$. An alternative and purely algebraic test condition was derived in [8] in form of a frequency-dependent eigenvalue problem. This result generalizes the approaches adopted in [9] for delayless models and in [10] for delayed models of transmission lines based on the Generalized Method of Characteristics (GMoC). The main theorem in [8] asserts that the imaginary eigenvalues of

$$ H(s)\xi = s\xi $$

correspond to the points where one of the singular values of $S(j\omega)$ crosses the unit threshold. Therefore, if there are no purely imaginary solutions to (5), the model is passive. The complex matrix $H(s)$ turns out to have Hamiltonian structure and is defined in terms of the state-space matrices of (3), see [8].

Figure 1 depicts the trajectory of one singular value $\sigma_i(j\omega)$ exceeding the unit threshold in the bandwidth $(\omega_{2k-1}, \omega_{2k})$, thus causing a localized passivity violation. In order to enforce passivity, two main perturbation approaches may be devised. According to [11], where a similar notation was used in the context of GMoC-based transmission line macromodels, we denote these two schemes as vertical and horizontal displacement, respectively. Both schemes attempt at removing the passivity violation by applying a small perturbation to (3)

$$ C_m \leftarrow C_m + \delta C_m, $$

using a different form of local passivity constraints. The following two sections describe the two approaches.

**Vertical displacement** In the vertical displacement scheme, we want to find a perturbation $\delta \delta_k$ such that

$$ \hat{\delta}_k + \delta \sigma_k < 1, $$

where $\hat{\delta}_k$ is the maximum value assumed by some $\sigma_k$ within a violation band. This perturbation is denoted with a vertical arrow in Fig. 1. Denoting the corresponding left and right singular vectors as $u_k$ and $v_k$, respectively, and performing a first-order singular value perturbation [15], we obtain

$$ \delta \sigma_k = \Re \left\{ \sum_{m=1}^M u_k H_m w_{k,m} \right\} $$

where

$$ w_{k,m} = e^{-j\omega_k \tau_m} (j\omega_k I - A)^{-1} B v_k. $$

Using the properties of the Kronecker product [13] we obtain

$$ \delta \sigma_k = \sum_{m=1}^M \Re \left\{ w_{k,m}^T \otimes u_k^H \right\} \text{vec}(\delta C_m), $$

where the vec operator stacks the columns of its matrix argument in a single vector. Finally, we collect all the constraints (for each $\delta_k$) and we obtain the matrix form of the vertical passivity constraint

$$ \hat{Z} \text{vec}(\delta C) \approx \hat{b}, $$

where $\delta C = [\delta C_1 \cdots \delta C_M]$, $\hat{Z} = \left[ \hat{Z}_{1,1} \cdots \hat{Z}_{1,M} \\ \vdots \ddots \vdots \\ \hat{Z}_{K,1} \cdots \hat{Z}_{K,M} \right]$, $\hat{b} = \left[ 1 - \hat{\sigma}_1 \cdots 1 - \hat{\sigma}_K \right]$, and

$$ \hat{Z}_{k,m} = \Re \{ w_{k,m}^T \otimes u_k^H \}. $$

**Horizontal displacement** Horizontal displacement aims at collapsing the passivity violation bands by perturbing their edges, as depicted by horizontal arrows in Fig. 1. Any of these edges $\delta_i = j\hat{\omega}_i$ corresponds to a purely imaginary eigenvalue of (5). Therefore, this displacement is easily obtained by applying a suitable perturbation $\delta H$ to the Hamiltonian matrix $H(s)$. The first-order relationship between the matrix and eigenvalue perturbation reads [11]

$$ j\hat{\omega}_{1,p} - j\hat{\omega}_{1} \simeq \frac{\xi_i^H J \delta H \xi_i}{\xi_i^H J (I - H'(s_i)) \xi_i}, $$

where $\xi_i$ is the eigenvector associated to $s_i$, $J = \begin{bmatrix} 0 & I \\ -I & 0 \end{bmatrix}$, and $H'(s_i)$ is the derivative of $H(s)$ with respect to $s$. The numerator in (14) can be evaluated as function of the perturbation matrices $\delta C_m$. A tedious but straightforward derivation leads to

$$ \hat{Z} \text{vec}(\delta C) \approx \hat{b} $$

Figure 1: Notation used in vertical and horizontal displacement algorithms.
where $\tilde{Z}$ is expressed in terms of the state-space matrices in (3) and

$$\tilde{b} = \begin{bmatrix} \Im \{ \xi_i^H I (I - H'(\tilde{s}_1)) \xi_1 \} (\tilde{\omega}_{1,p} - \tilde{\omega}_1) \\ \vdots \\ \Im \{ \xi_i^H I (I - H'(\tilde{s}_1)) \xi_i \} (\tilde{\omega}_{i,p} - \tilde{\omega}_1) \end{bmatrix}. \tag{17}$$

**Accuracy preservation** Our proposed passivity compensation schemes amount to minimizing some norm $||\text{vec}(\delta C)||$ of the model perturbation while enforcing the constraints (11) or (16). This section details how to select the optimal norm. In order to preserve the accuracy of the model, we minimize the energy of the induced perturbation on the impulse response

$$E = \sum_{p,q=1}^P \int_0^\infty |\delta h_{p,q}(t)|^2 \, dt = \int_0^\infty \text{tr}(\delta h(t)\delta h(t)^T) \, dt. \tag{18}$$

An alternative and purely algebraic characterization of this impulse perturbation energy is obtained as illustrated in [12], obtaining

$$E = \text{tr}(\delta C W \delta C^T) = \text{tr}(\delta C K^T K \delta C^T) = \text{tr}(\delta C K \delta C^T) = ||\text{vec}(\delta C_K)||^2, \tag{19}$$

where $W = K^T K$ is obtained by the solution of delay-dependent and small-size Lyapunov equations. This characterization shows that the minimal perturbation in the model response is achieved by minimizing a weighted norm of $\delta C$. In our implementation, this weight is simply applied as a basis change defined by the Cholesky factor $K$

$$\text{vec}(\delta C) = \text{vec}(\delta C_KK^{-T}) = (K^{-1} \otimes I) \text{vec}(\delta C_K). \tag{20}$$

In summary, we formulate our proposed passivity enforcement as

$$\begin{array}{ll}
\text{minimize} & ||\text{vec}(\delta C_K)|| \\
\text{s.t.} & \tilde{Z}(K^{-1} \otimes I) \text{vec}(\delta C_K) < \tilde{b} \tag{21}
\end{array}$$

for the vertical displacement algorithm and

$$\begin{array}{ll}
\text{minimize} & ||\text{vec}(\delta C_K)|| \\
\text{s.t.} & \tilde{Z}(K^{-1} \otimes I) \text{vec}(\delta C_K) = \tilde{b} \tag{22}
\end{array}$$

for the horizontal displacement algorithm. Problem (21) is solved using standard pseudoinverse methods, whereas (22) is solved using an interior-point convex optimization method [14]. Iterative application of (21) or (22) achieves passivity in few steps, as illustrated by the numerical examples.

**Examples**

The first example we consider is a 10-cm interconnect on a Printed Circuit Board (PCB). The structure includes a stripline and perturbed passive models.
vertical displacement algorithm required only 2 iterations and a CPU time of 1.8 seconds, whereas the horizontal displacement method required 2 iterations and 5.5 seconds. Figure 2 depicts the singular values trajectories around the passivity violation bands before and after passivity enforcement. As shown in Fig. 3, the accuracy of the model is well-preserved by both algorithms (relative errors of $5.546 \times 10^{-3}$ and $8.291 \times 10^{-3}$ for vertical and horizontal displacement schemes, respectively).

The second example considers a test vehicle that was specifically designed to assess the quality of multi-chip transmission at various bit rates. The hardware includes a backplane and a daughter card connected by a high-performance connector. Three adjacent signal lines are considered in this analysis. The corresponding 6-port interconnect structure was measured with a VNA from nearly DC up to 20 GHz, resulting in a total of 2000 frequency points. This dataset was processed by a DVF algorithm, leading to a delayed rational macromodel with 15 poles for each scattering parameter and a variable number of delays (between 1 and 3).

A small passivity violation was detected in the model, with a maximum singular value equal to 1.015 located around the DC point, see Fig. 4. The vertical algorithm was applied in order to enforce the passivity, requiring 2 iterations and 6.1 seconds (the horizontal scheme produced similar results, not shown). The singular values of original non-passive and perturbed passive model are depicted in Fig. 4.

An equivalent circuit in form of a SPICE netlist was synthesized from the passive macromodel. A number of SPICE runs was performed using realistic models for driver and receivers, in order to assess the maximum bit rate that the channel can support. To this end, a pseudo-random bit stream of 4000 bits was launched on one of the lines, and two synchronous clock signals were applied to the two aggressor lines. The eye diagrams depicted in Fig. 5 and corresponding to bit rates of 1, 2, and 3 Gbps were obtained in less than 15 minutes using a standard laptop and a freeware SPICE engine [16]. These results demonstrate the feasibility of our macromodeling approach, that is able to deliver fast and accurate channel qualification results via simple circuit-based simulation.

References


[16] LTSPICE IV, Linear Technology, available online: www.linear.com